# Introduction to Astronomy and Astrophysics - 1

## IUCAA-NCRA Graduate School 2013

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IUCAA

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IUCAAAAIntro12013DB

# Coordinate Systems, Units and the Solar System

# Locating Objects

- Angular position can be measured accurately; distance difficult
- Spherical Polar Coordinate System:

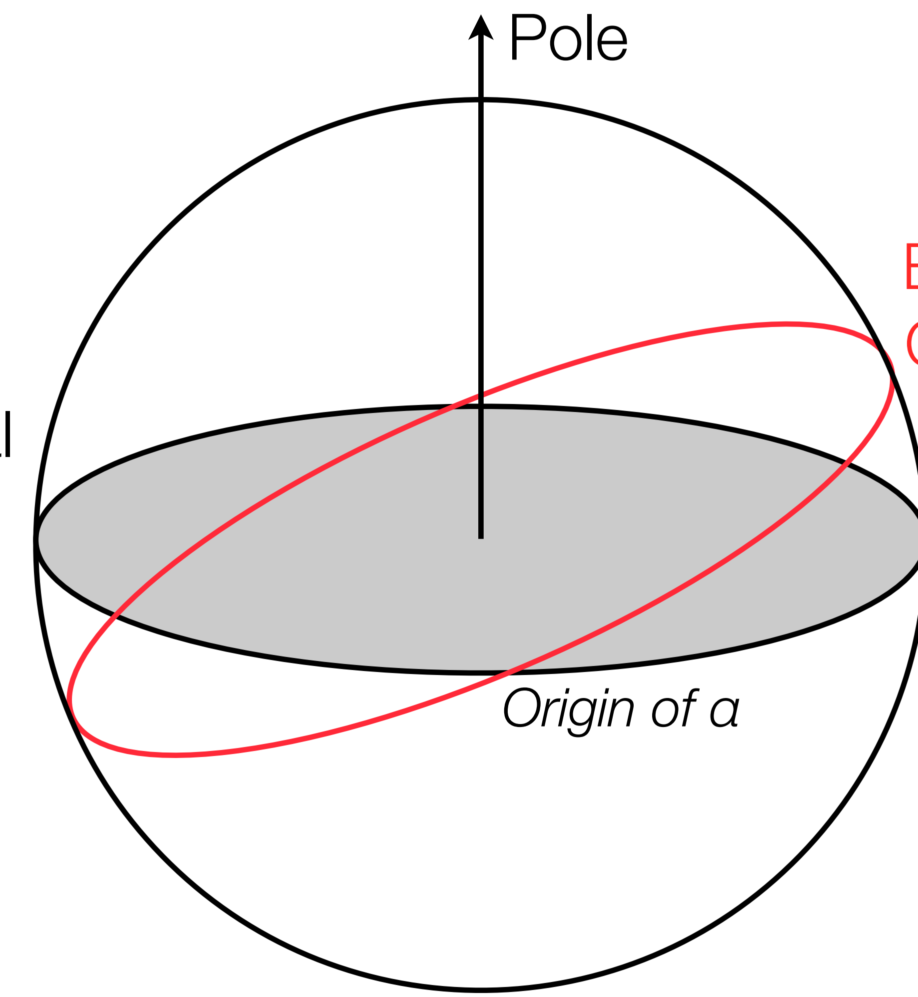
  latitude = Declination ($\delta$)
  longitude = Right Ascension ($\alpha$)

## Time

Solar Day = 24 h
(*time between successive solar transits*)

Earth's Spin Period: 23h56m
(*time between successive stellar transits*)

**24h "Sidereal Time" = 23h56m Solar Time**

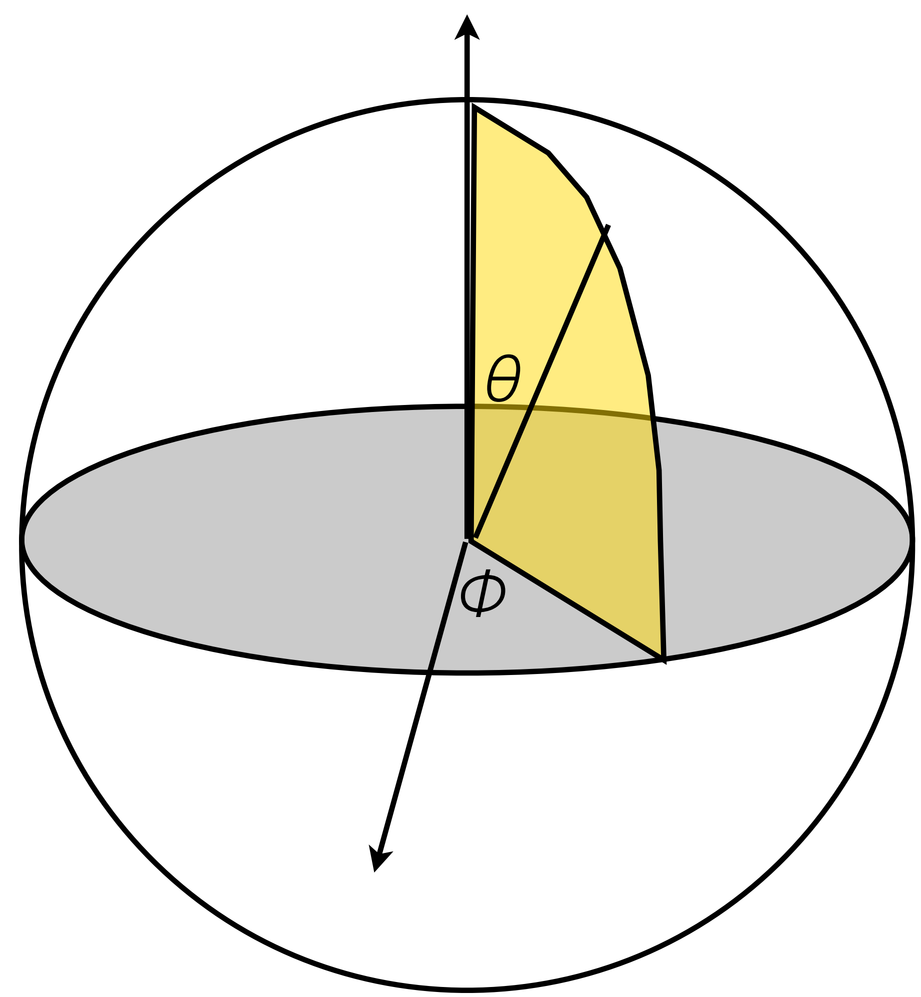



*α is expressed in units of time*
*Transit time of a given α = Local Sidereal Time*

# Different coordinate systems

**Convention:** *Latitude: ($\pi/2 - \theta$) ; Longitude: $\Phi$*

- **Equatorial:** Poles: extension of the earth's spin axis
- **Ecliptic:** Poles: Normal to the earth's orbit around the sun
- **Galactic:** Poles: Normal to the plane of the Galaxy




For Equatorial and Ecliptic: same longitude reference (ascending node - vernal equinox)

For Galactic coordinates: longitude reference is the direction to the Galactic Centre

Equatorial coordinates: larger $\Phi$, later rise: RA ($\alpha$)
latitude = Declination ($\delta$)

# Rise and Set

**Horizon:** tangent plane to the earth's surface at observer's location

geographic latitude λ

$OP = \sin\delta$ ; $QP = \cos\delta = PB$

$AP = OP \tan\lambda = \sin\delta \tan\lambda$

$\angle APB = \cos^{-1}(AP/PB) = \cos^{-1}\{\tan\delta \tan\lambda\}$

Total angle spent by the source above the horizon

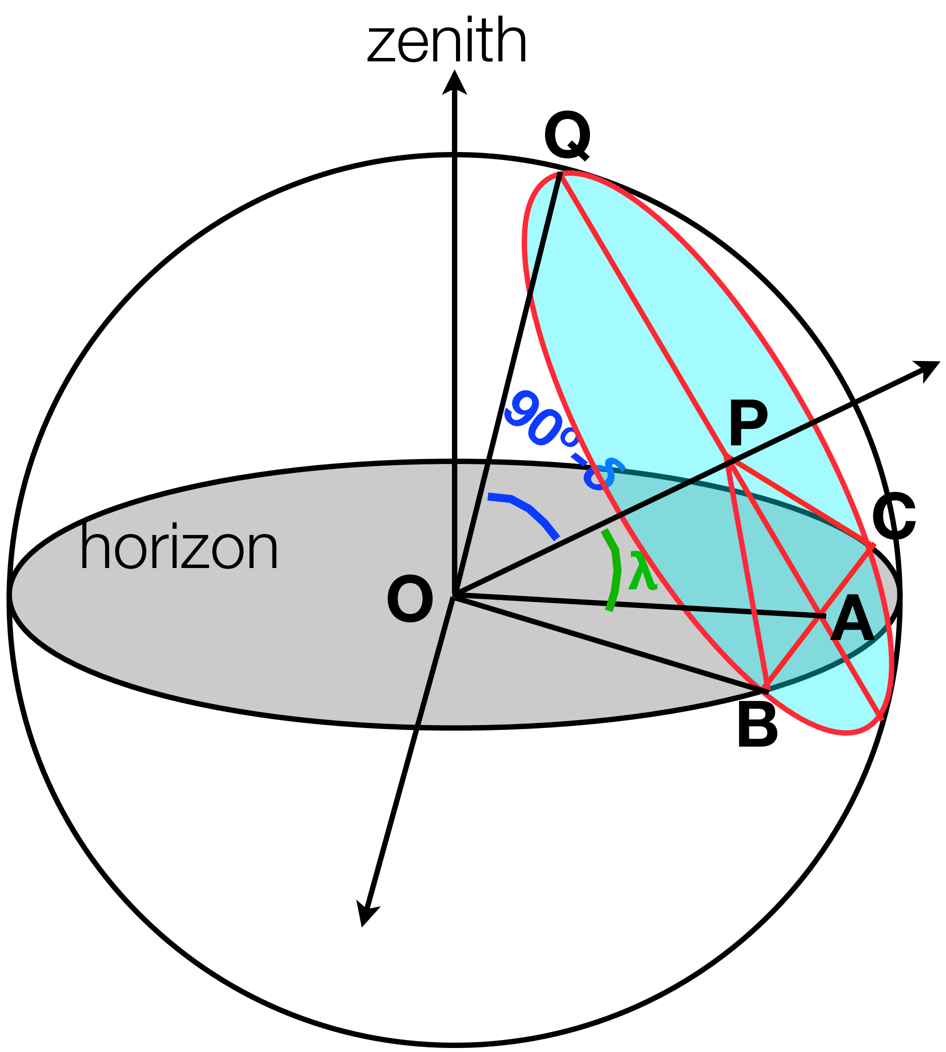
$= 360^o - 2\cos^{-1}\{\tan\delta \tan\lambda\}$

Time spent by the source above the horizon

$= ([360^o - 2\cos^{-1}\{\tan\delta \tan\lambda\}]/15)$ sidereal hours

$= (1436/1440)\ ([360^o - 2\cos^{-1}\{\tan\delta \tan\lambda\}]/15)$

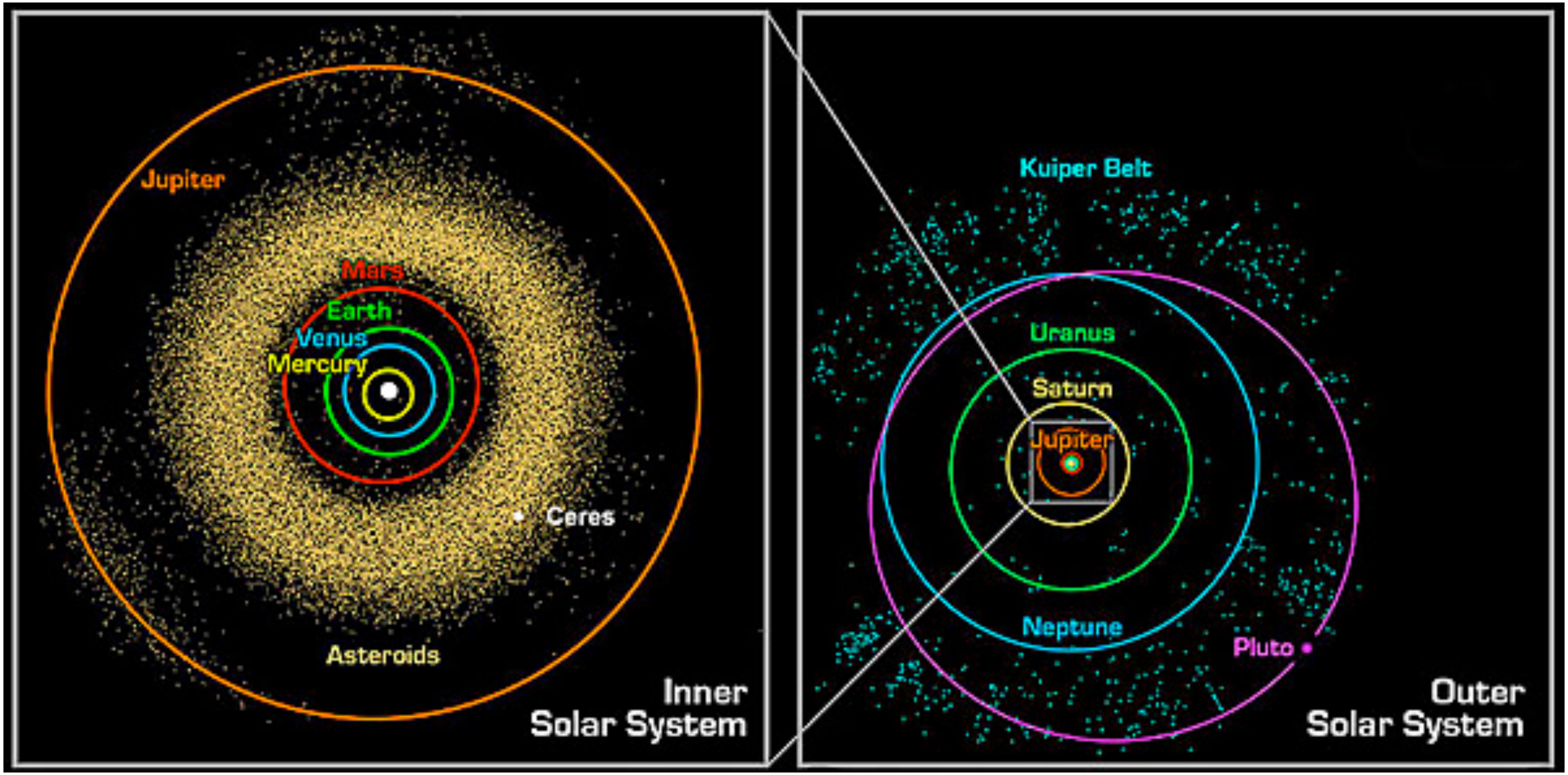
hours by solar clock



# Earth-Moon system

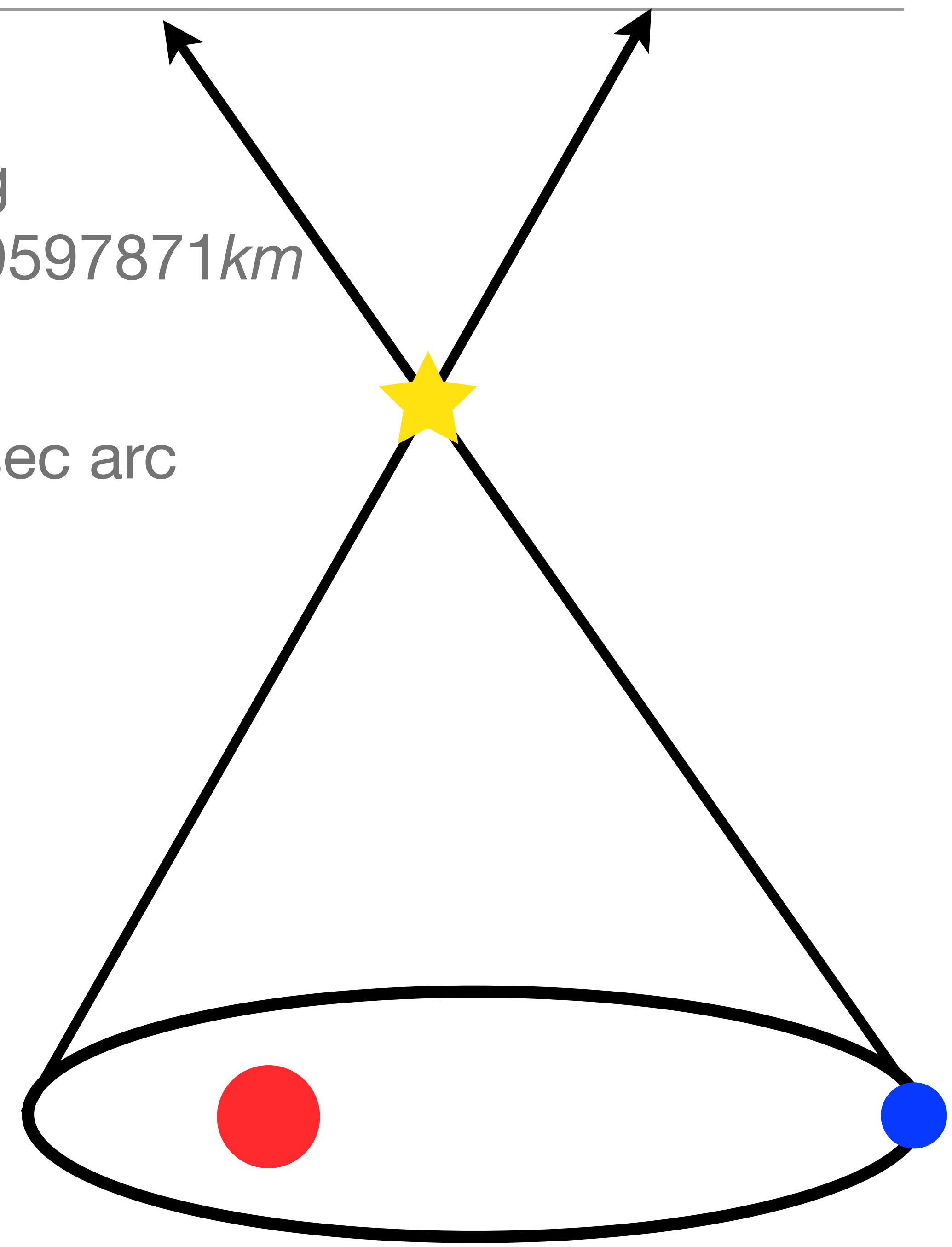
- Tidally locked. Moon's spin Period = Period of revolution around the Earth
- $M_{earth}$ = 5.97722 x $10^{24}$ kg ; $M_{moon}$ = 7.3477 x $10^{22}$ kg
- Orbital eccentricity = 0.0549
- Semi-major axis = 384,399 km, *increasing by 38 mm/y* (1ppb/y)
  Earth's spin angular momentum being pumped into the orbit
- Origin of the moon possibly in a giant impact on earth by a mars-sized body; moon has been receding since formation.
- At present the interval between two new moons = 29.53 days
- Moon's orbital plane inclined at 5.14 deg w.r.t. the ecliptic
- Earth around the sun, Moon around the earth: same sense of revolution
- Moon is responsible for total solar eclipse as angular size of the sun and the moon are roughly similar as seen from the earth.

# Bodies in The Solar System




***A. Feild (STSCI)***

# Measuring Distance



- Inside solar system: Radio and Laser Ranging
  *Earth-Sun distance*: 1 Astronomical Unit = 149597871*km*
- Nearby stars: Parallax
  Parsec = distance at which 1 AU subtends 1 sec arc
  = $3.086 \times 10^{13}$ *km* = 3.26 light yr
- Distant Objects: Standard Candles
  - Cepheid and RR Lyrae stars
  - Type Ia Supernovae
- Cosmological distance: Redshift

Linear size = angular size x distance

# Measure of Intensity

1 Jansky = $10^{-26}$ W/m$^2$/Hz

Optical Magnitude Scale

Logarithmic Scale of Intensity: $m = -2.5 \log (I/I_0)$ *apparent magnitude*

Absolute Magnitude (measure of luminosity):

$$M = -2.5 \log (I_{10pc} / I_0)$$

The scale factor $I_0$ depends on the waveband, for example:

| **Band** | $I_0$ |
|---|---|
| Johnson U | 1920 Jy |
| B | 4130 |
| V | 3690 |
| R | 3170 |
| I | 2550 |

# References

- The Physical Universe : *Frank H. Shu*
- An Introduction to Modern Astrophysics : *B.W. Carroll & D.A. Ostlie*
- Astrophysical Quantities: *C.W. Allen*
- Astrophysical Formulae: *K.R. Lang*

# Orbits

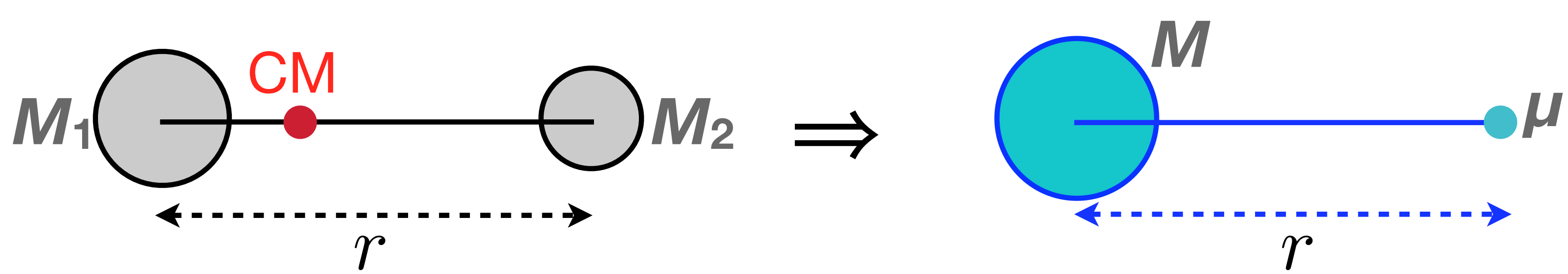
# Kepler Orbit

1. Elliptical orbit with Sun at one focus
2. Areal velocity = constant
3. $T^2 \propto R^3$

*Conserved Energy* $E$
*and Angular Momentum* $J$

Dynamical time scale in gravity: $$\tau \approx \frac{1}{\sqrt{G\rho}}$$

Two body problem:




$$r = \frac{l}{1 + e\cos\phi}\;;\quad l = \frac{J^2}{GM\mu^2}\;;\quad e = \left[1 + \frac{2EJ^2}{G^2M^2\mu^3}\right]^{1/2}$$

$$M = M_1 + M_2$$

$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

$E < 0 \implies e < 1$; bound elliptical orbit:

$$a = \frac{GM\mu}{2|E|}\;;\quad J = \mu\sqrt{GMa(1-e^2)}$$

# Motion in a Central Force Field

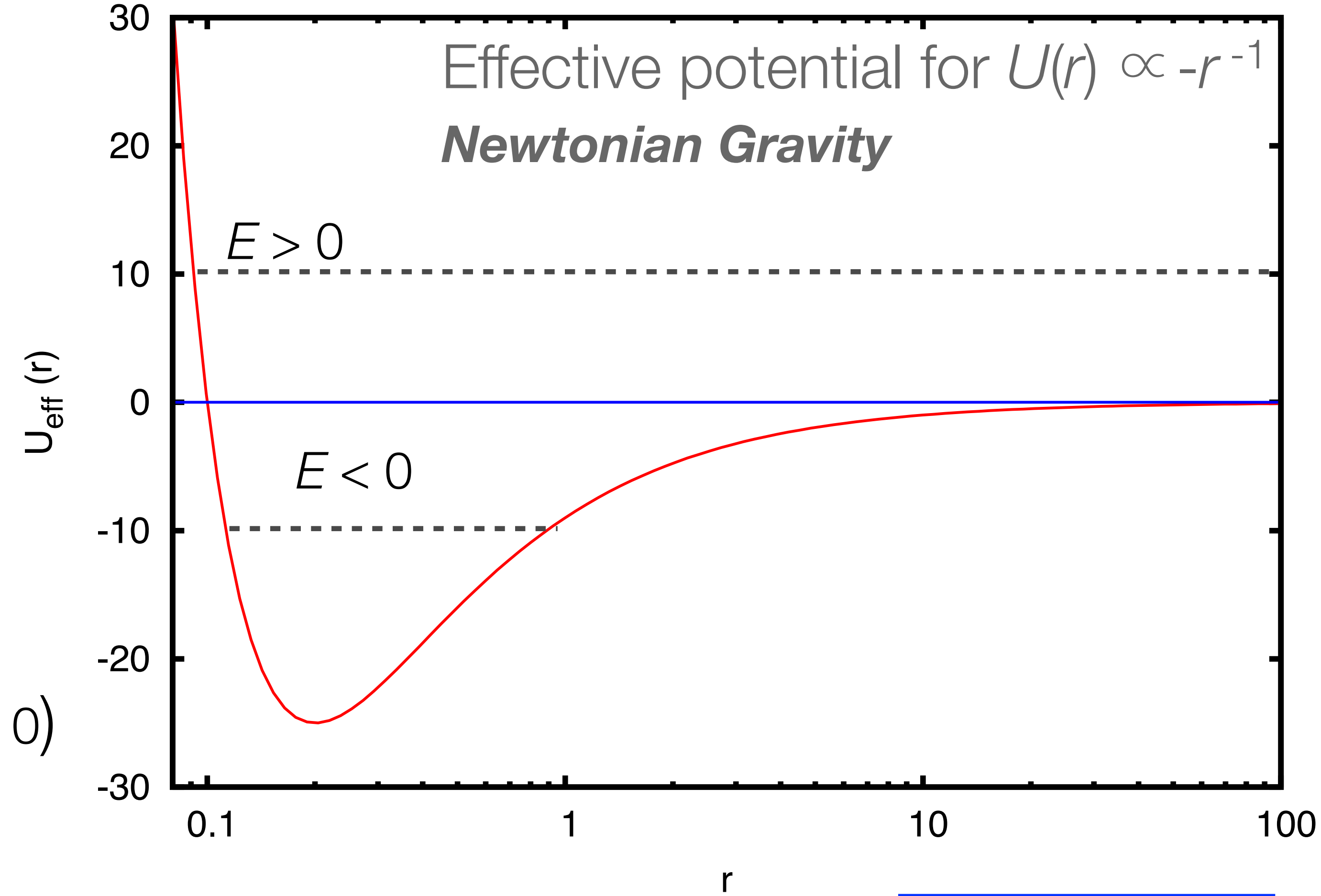
Energy $E = \frac{1}{2}\mu\dot{r}^2 + \boxed{\frac{J^2}{2\mu r^2} + U(r)}$ ; $\dot{r}^2 = \frac{2}{\mu}\left[E - U_{\rm eff}(r)\right]$

$U_{\rm eff}(r)$

$$\phi = \int \frac{(J/r^2)dr}{\left[2\mu\left\{E - U_{\rm eff}(r)\right\}\right]^{1/2}} + {\rm const.}$$

Intersections of $U_{\rm eff}(r)$ with $E$ give turning points in the orbit

Newtonian Gravity: $U(r) = -\frac{GM\mu}{r}$




Setting $U_{\rm eff} = E$ gives turning points:

$$\frac{1}{r_{\substack{\rm min\\ \rm max}}} = \frac{GM\mu^2}{J^2}\left[1 \pm \sqrt{1 + \frac{2J^2E}{G^2M^2\mu^3}}\right]$$

and for $E < 0$ ($r_{\rm max}$ does not exist for $E > 0$)

$$\Delta\phi = 2\left[\phi(r_{\rm max}) - \phi(r_{\rm min})\right] = 2\pi$$

*i.e.* orbit is closed. Departure from *1/r* or $r^2$ potential give $\Delta\phi \neq 2\pi$ $\Rightarrow$ **precession of periastron**

# Departure from $1/r^2$ gravity

Common causes of departure from 1/r form of gravitational potential:

- Distributed mass
- Tidal forces
- Relativistic effects

$$U = -\frac{\alpha}{r} + \frac{\beta}{r^2} \Rightarrow \underset{\text{per orbit}}{\delta\phi} = -2\pi\beta\mu/J^2$$

$$U = -\frac{\alpha}{r} + \frac{\gamma}{r^3} \Rightarrow \underset{\text{per orbit}}{\delta\phi} = -6\pi\alpha\gamma\mu^2/J^4$$

In relativity, effective potential near a point mass (including rest energy)

$$\bar{E} = \left[\left(1 - \frac{1}{\bar{r}}\right)\left(1 + \frac{\bar{a}^2}{\bar{r}^2}\right)\right]^{1/2}$$

$$\bar{E} = E/mc^2$$
$$\bar{a} = J/mcr_g$$
$$r_g = 2GM/c^2$$

Newtonian approximation $\bar{r} \gg 1$

Next order correction, upon expanding the square root: $-\frac{\bar{a}^2}{2\bar{r}^3}$

Gives $\underset{\text{per orbit}}{\delta\phi} = \frac{6\pi GM}{a(1-e^2)c^2}$ $\Rightarrow$ Precession of perihelion of Mercury

# Schwarzschild Gravity: Equation of Motion & Effective Potential

$$\left(\frac{1}{1-1/\bar{r}}\right)\left(\frac{d\bar{r}}{d\tau}\right)^2 = \frac{1}{\bar{E}^2}\left[\bar{E}^2 - 1 + \frac{1}{\bar{r}} - \frac{\bar{a}^2}{\bar{r}^2} + \frac{\bar{a}^2}{\bar{r}^3}\right]$$

Effective potential by setting LHS = 0

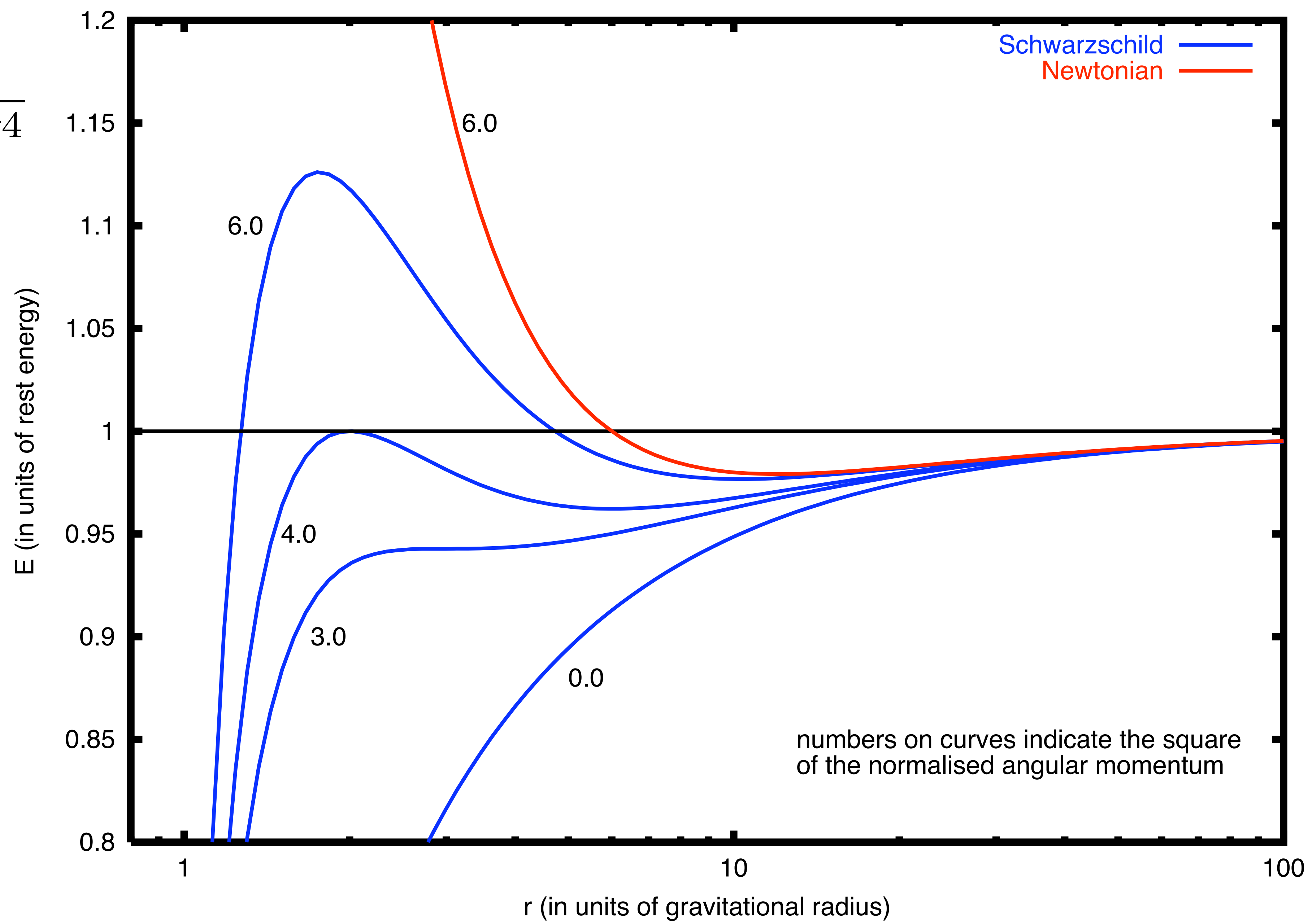
$$\left(\frac{1}{1-1/\bar{r}}\right)\left(\frac{d\phi}{d\tau}\right)^2 = \frac{\bar{a}^2}{\bar{E}^2\bar{r}^4}$$

A binary orbit decays due to Gravitational Wave radiation

$$-\frac{dE}{dt} = \frac{32}{5}\frac{G^4}{c^5a^5}M_1^2M_2^2 \times (M_1+M_2)f(e)$$

$$\frac{da}{dt} = \frac{2a^2}{GM_1M_2}\frac{dE}{dt}$$

$$\frac{de}{dt} = (1-e)\frac{1}{a}\frac{da}{dt}$$



# Photon Orbit in Schwarzschild Gravity

setting $m = 0,\ \bar{E} \to \infty,\ \bar{a} \to \infty,\ \frac{\bar{a}}{\bar{E}} \to \frac{b}{r_g} \equiv \bar{b}$ $\qquad b =$ impact parameter at $\infty$

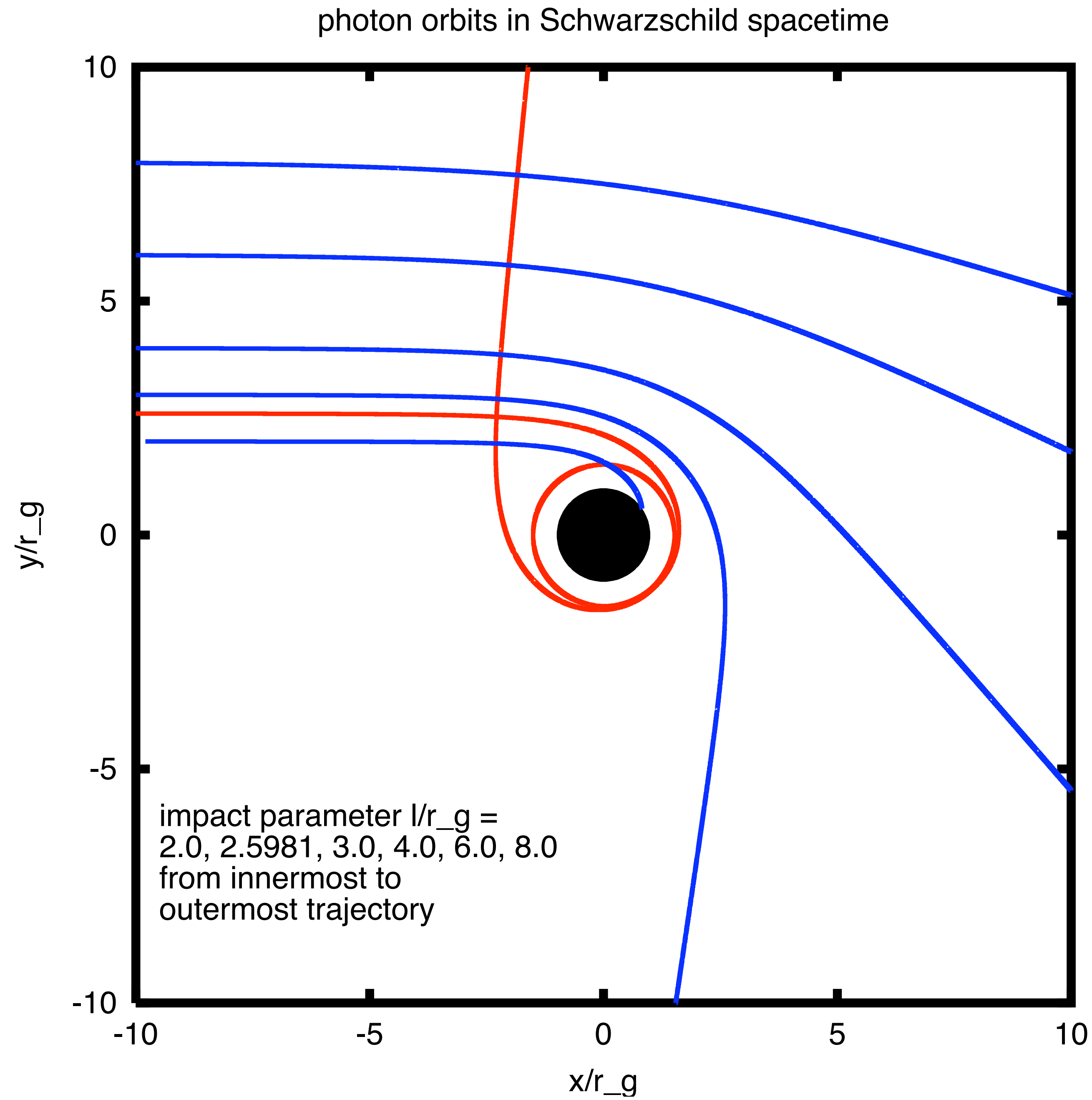
$$\left(\frac{1}{1-1/\bar{r}}\right)\left(\frac{d\bar{r}}{d\tau}\right)^2 = 1 - \frac{\bar{b}^2}{\bar{r}^2} + \frac{\bar{b}^2}{\bar{r}^3}$$

$$\left(\frac{1}{1-1/\bar{r}}\right)\left(\frac{d\phi}{d\tau}\right)^2 = \frac{\bar{b}^2}{\bar{r}^4}$$

hence

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{b^2}\left(1 - \frac{b^2}{r^2} + r_g\frac{b^2}{r^3}\right)$$

integrate to get orbit.
3rd term on RHS causes
curvature of light path




Gravitational Lensing

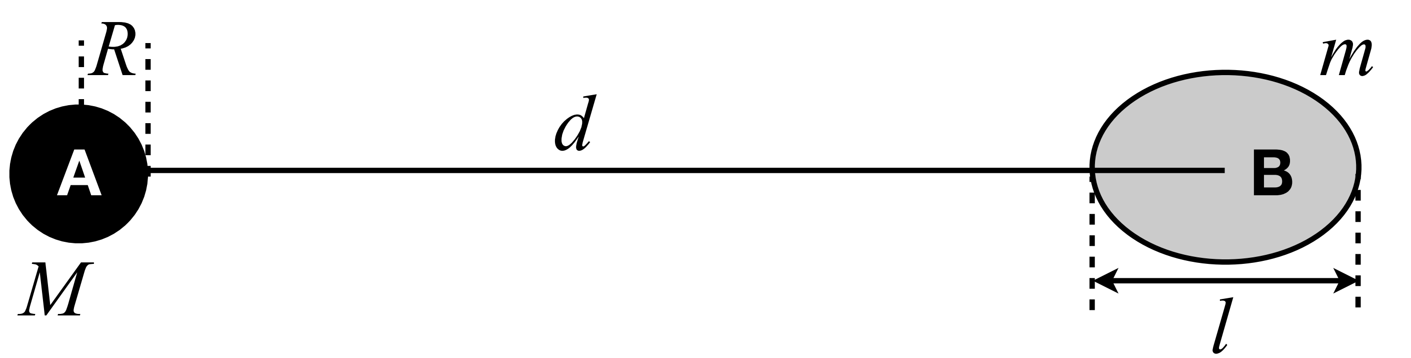
# References

- Classical Mechanics : *H. Goldstein*
- Relativistic Astrophysics : *Ya B. Zeldovich & I.D. Novikov*
- Gravitation : *C. Misner, K.S. Thorne & J.A. Wheeler*
- Classical Theory of Fields : *L.D. Landau & E.M. Lifshitz*

# Tidal forces and Roche Potential

# Tidal effect

Gradient of external gravitational force across an extended body tends to deform the object - responsible for tides on Earth




Tidal Force: $F_T = \dfrac{2GMm}{d^3} l$

Self gravity of object B: $F_g = \dfrac{Gm^2}{l^2}$

Object B would not remain intact if $F_T > F_g$

$\therefore$ condition for stability: $l^3 < \dfrac{m}{2M} d^3$ , or $\dfrac{l}{d} < \left(\dfrac{q}{2}\right)^{1/3}$ , $q \equiv \dfrac{m}{M}$

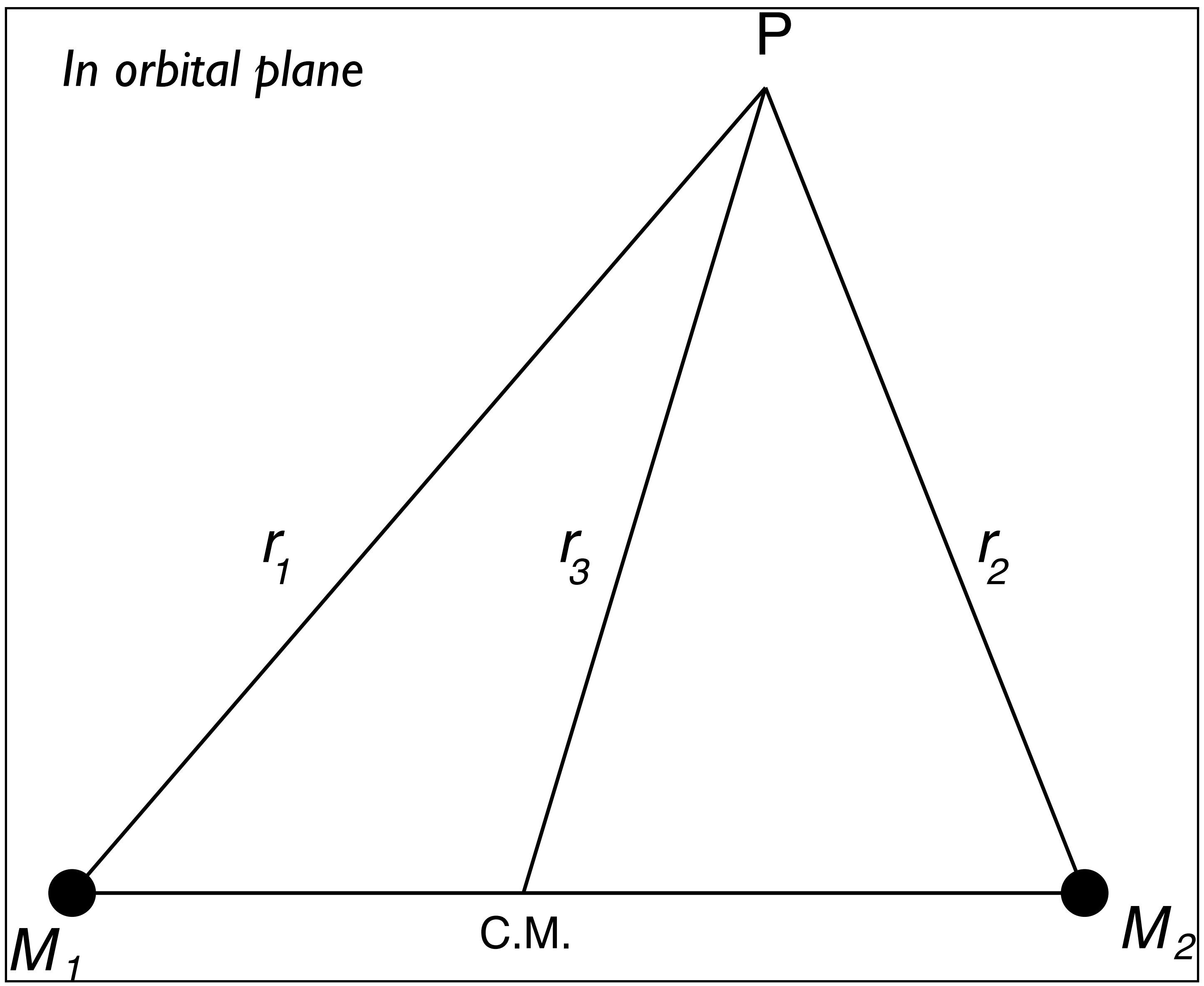
Disruption would occur if $d^3 < 2\dfrac{l^3}{m} M = 2R^3 \left(\dfrac{M}{4\pi R^3/3}\right) \left(\dfrac{m}{8 \times 4\pi (l/2)^3/3}\right)^{-1}$

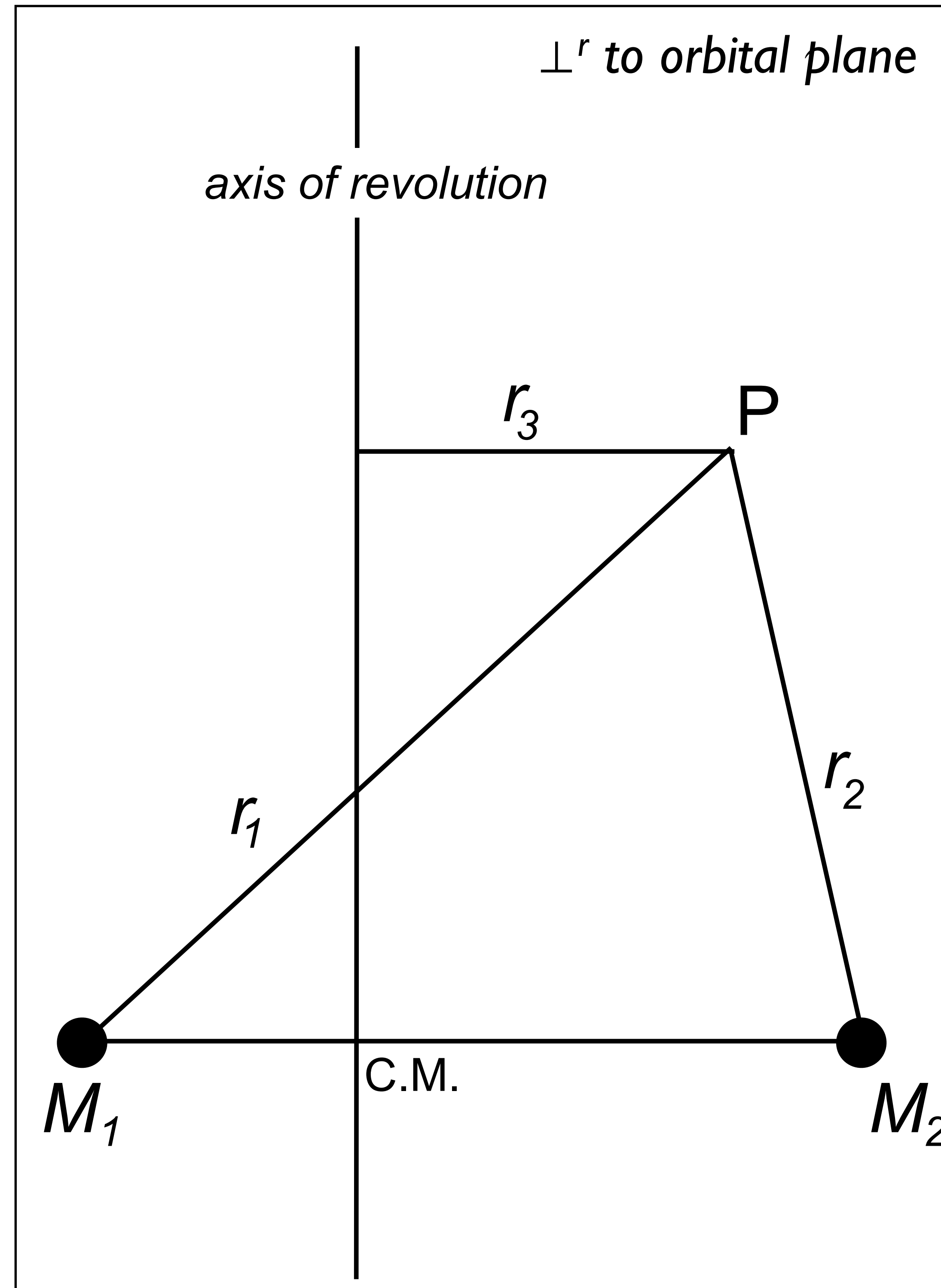
i.e. for $d < 2^{4/3} R \left(\dfrac{\rho_M}{\rho_m}\right)^{1/3}$ **: Roche limit**

# Roche Potential in a binary system

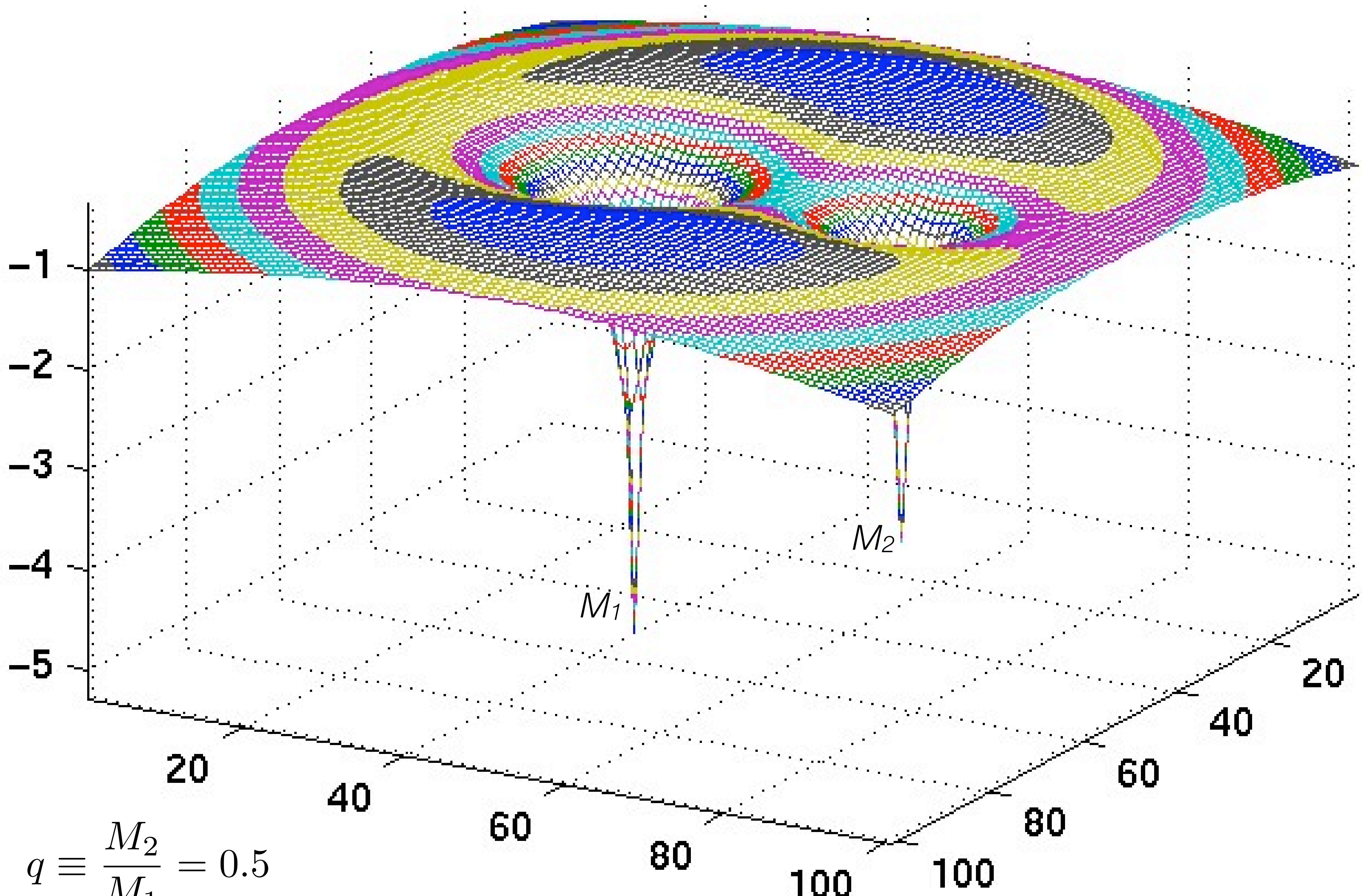
The surface of a fluid body is an equipotential

In corotating frame $V = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{\Omega^2 r_3^2}{2}$



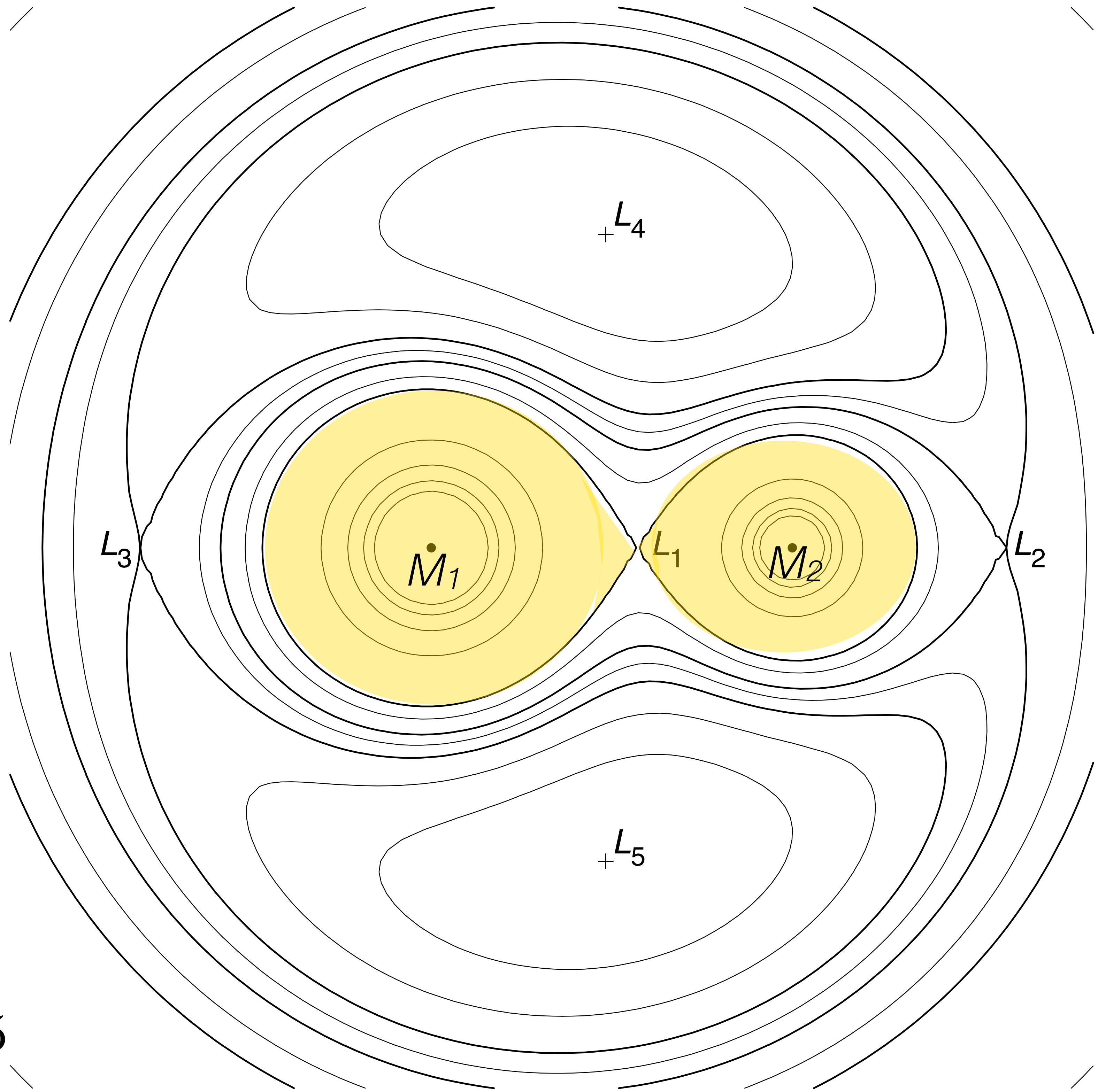
# Roche Potential in the equatorial plane




$$q \equiv \frac{M_2}{M_1} = 0.5$$

# Lagrangian points and the Roche Lobe

*In orbital plane*




A star overfilling its Roche Lobe would transfer matter to its companion

$$q \equiv \frac{M_2}{M_1} = 0.5$$

# References


- Theoretical Astrophysics, vol. 1 sec. 2.3 : *T. Padmanabhan*
- Close Binary Systems : *Z. Kopal*
- Hydrodynamic and Hydromagnetic Stability: *S. Chandrasekhar*
- Astrophysical Journal vol. 55, p. 551 (1984) ; *S.W. Mochnacki*

# Hydrostatic Equilibrium

# Equations of Fluid Mechanics

Continuity Equation: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

Euler Equation: $\rho \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right] = -\vec{\nabla} P + \rho \vec{g}$

Equation of State: $P = P(\rho)$

*The fluid is described by the quantities*

$$\rho, \ P, \ \vec{v}$$

*that are functions of space and time.*

*Viscosity is ignored*

Stationarity follows by setting time derivatives $\frac{\partial}{\partial t}$ to zero

Hydrostatic equilibrium follows by setting both $\frac{\partial}{\partial t}$ and $\vec{v}$ to zero:

$$\vec{\nabla} P = \rho \vec{g}$$

# Hydrostatic Equilibrium

$$\vec{\nabla} P = \rho \vec{g}$$

In spherical symmetry:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

In relativity: (*Tolman, Oppenheimer, Volkoff*)

$$\frac{dP}{dr} = -\frac{G\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]\left[\rho(r) + \frac{P(r)}{c^2}\right]}{r^2\left(1 - \frac{2GM(r)}{c^2 r}\right)}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

Supplement with appropriate equation of state and solve for the structure of self-gravitating configurations such as stars, planets etc

# Virial Theorem

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$ (hydrostatic equilibrium)

Multiply both sides by $4\pi r^3$ and integrate over the full configuration: $r = 0 \to R$

$$4\pi R^3 P(R) - \int_0^R 4\pi r^2 \cdot 3P dr = \int_0^R 4\pi r^2 dr \left[-\frac{GM(r)\rho(r)}{r}\right]$$

RHS = Total gravitational energy of the configuration $E_g$ ( < 0 )

and since $P = (\gamma - 1)u_{\rm th}$ , where $u_{\rm th}$ is the Thermal (kinetic) energy density,

the 2nd term in LHS = $3(\gamma - 1)E_{\rm th}$ , $E_{\rm th}$ being the total thermal (kinetic) energy

Hence $E_g + 3(\gamma - 1)E_{\rm th} = 4\pi R^3 P(R)$ : **Virial Theorem**

must be obeyed by all systems in hydrostatic equilibrium

For $\gamma = 5/3$ and $P(R) = 0$ : $E_g + 2E_{\rm th} = 0$

Note: $E_{\rm tot} = E_g + E_{\rm th} = E_g/2 = -E_{\rm th}$

# A Rough Guide to Stellar Structure

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

stellar radius $R$

central pressure $P_{\rm c}$

total mass $M$

Using a linear approximation: $\frac{0 - P_{\rm c}}{R} = -\frac{GM}{R^2} \cdot \frac{M}{\frac{4\pi}{3} R^3}$

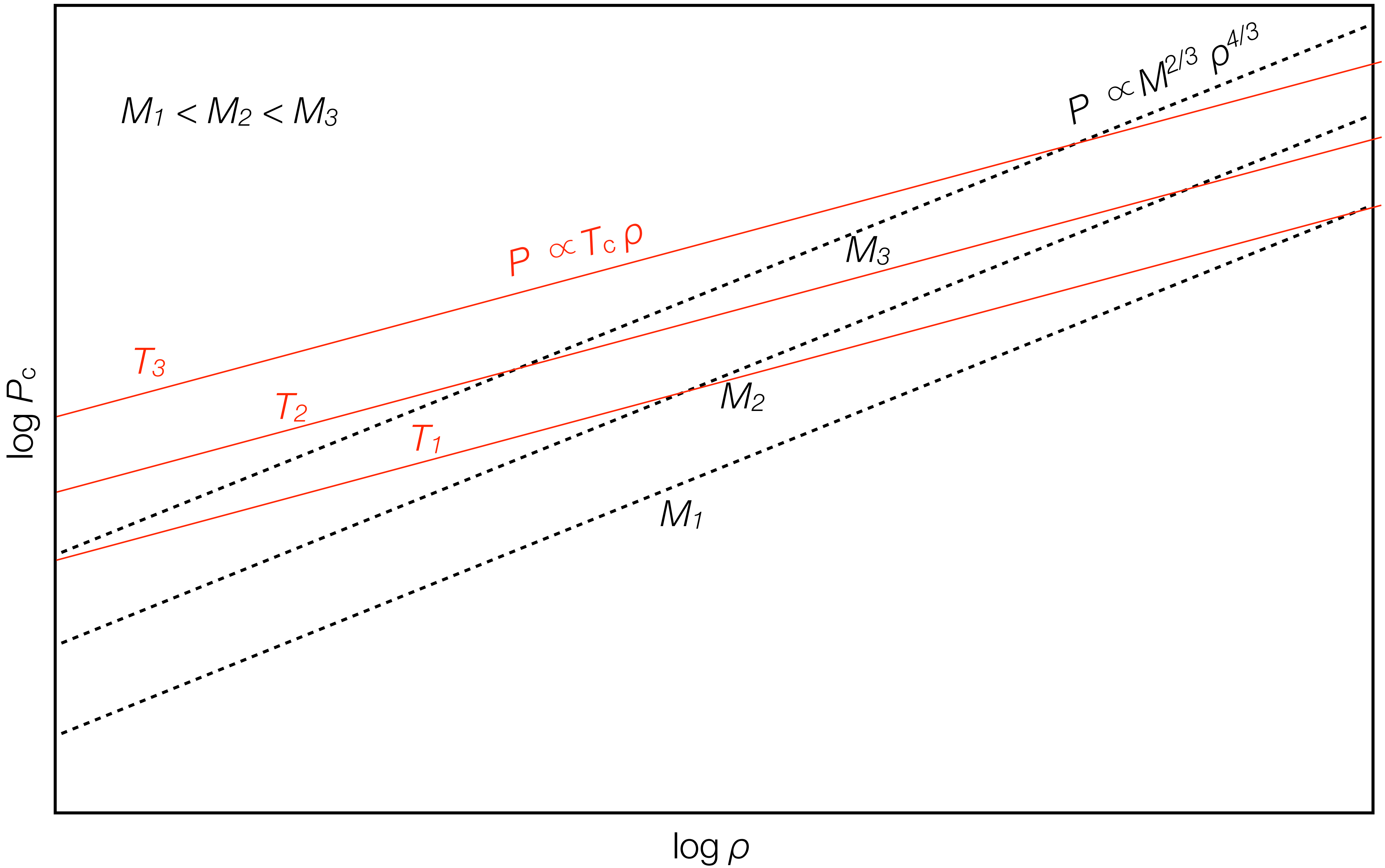
$$P_{\rm c} = \frac{3}{4\pi}\frac{GM^2}{R^4} = \left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\rho^{4/3}$$

(“gravitational pressure” $P_{\rm grav}$)

For equilibrium, this pressure needs to be matched by the Equation of State

e.g. Thermal Pressure: $P = \frac{kT}{\mu m_{\rm p}}\rho$

# Thermal pressure support



# References


- Astrophysics I : Stars : *R.L. Bowers*
- The Physical Universe : *F.H. Shu*
- Stellar Structure and Evolution : *R. Kippenhahn and A. Weigert*
- Physics of Fluids and Plasmas : *A. Rai Choudhuri*

# Stars

# Main Sequence

Hydrogen burning star $T_c \approx T_H$

$$P_c \approx GM^{2/3}\rho^{4/3} = \frac{kT_H}{\mu m_p}\rho \quad : \quad \rho \propto M^{-2} \; ; \; R \propto M \text{ and } T_c \propto \frac{M}{R}$$

Luminosity $L =$ Radiative Energy Content / Radiation Escape Time

Radiation Escape Time = $\left(\frac{R}{l}\right)^2 \cdot \frac{l}{c}$ where $l =$ mean free path $= \frac{1}{n\sigma} = \frac{1}{\rho\kappa}$ (opacity: $\kappa$)

Hence $L \approx \frac{aT^4R^3}{(R^2/lc)} \approx aclT^4R$ In Main Sequence: $L \propto lR \propto lM$

High mass stars: opacity: Thomson scattering $\kappa =$ constant; $l \propto \frac{R^3}{M}$ : $L_{MS} \propto M^3$

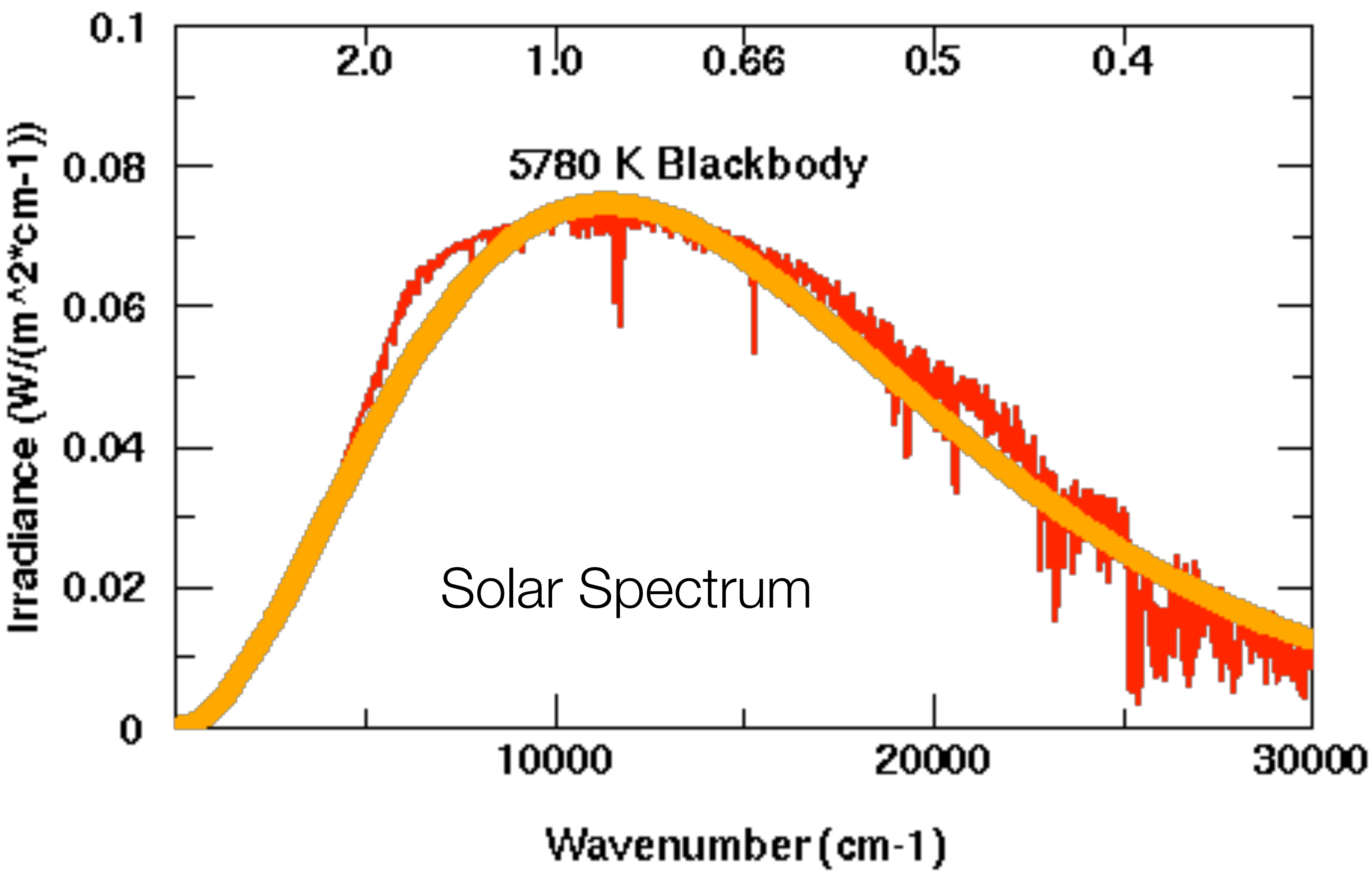
Low mass stars: $l \propto \frac{T^{3.5}}{\rho^2}$ : $L_{MS} \propto M^5$

*On average*

$$L_{MS} \propto M^4$$
$$t_{MS} \propto M^{-3}$$

# Effective Temperature

A typical stellar spectrum is nearly a blackbody




Color Temperature = Temp. of best-fit Planck Function

Total Luminosity

$$L = 4\pi\sigma T_{\rm eff}^4 R^2$$

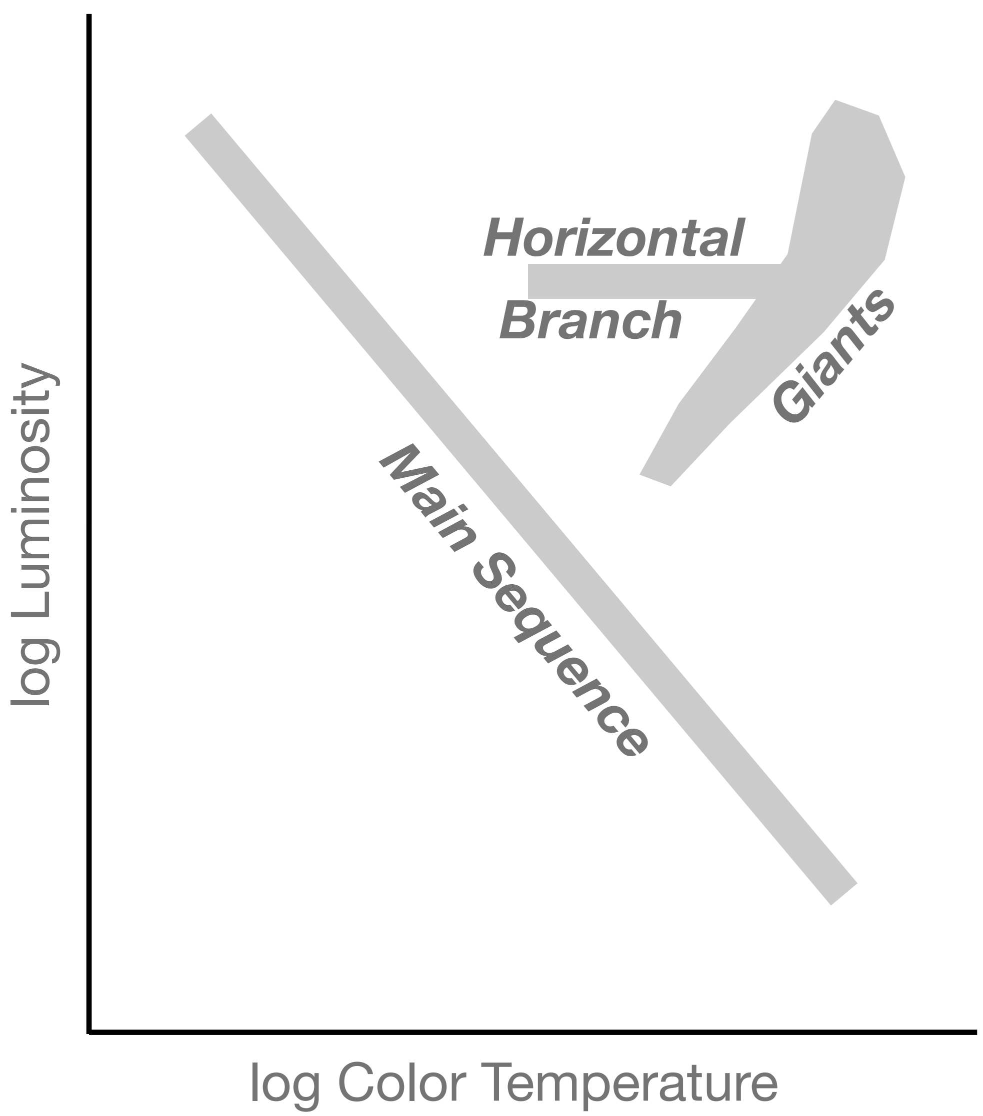
$T_{\rm eff}$ is defined as the Effective Temperature

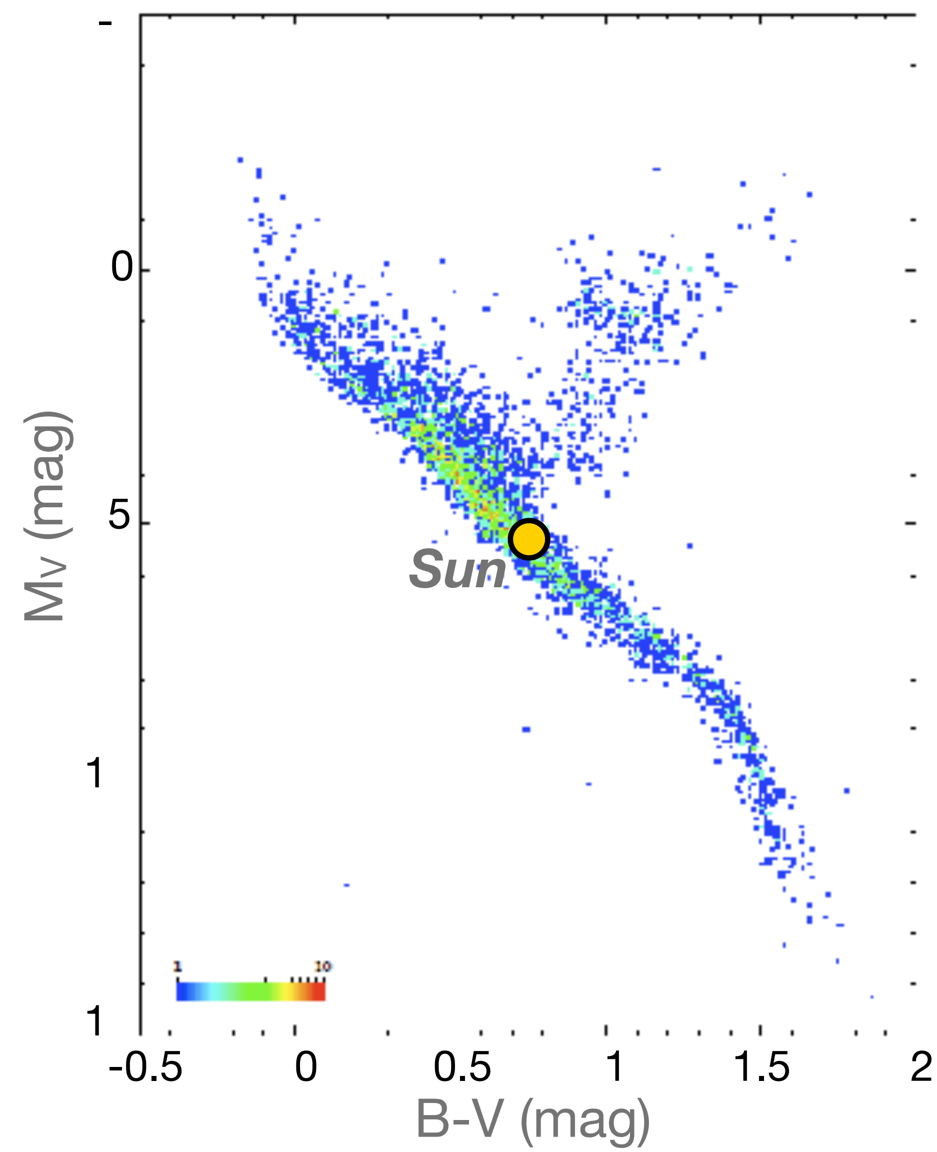
# Hertzsprung Russell Diagram for Stars

A plot of Luminosity vs. Temperature
(Absolute Magnitude vs. Color)

Near Blackbody spectrum:
$T_{\rm eff} \approx T_{\rm col}$

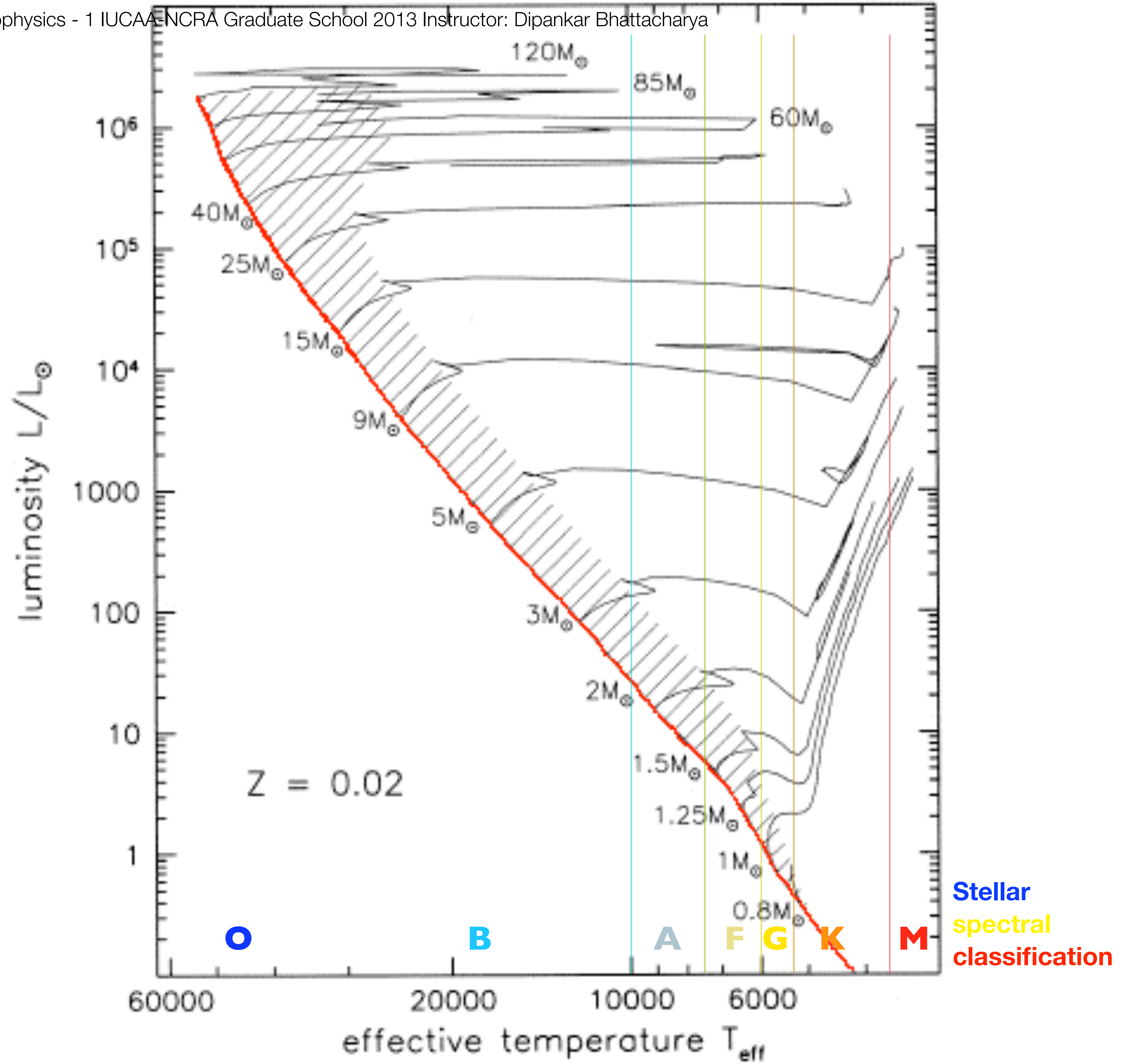
On Main Sequence $L \propto M^4$ ; $R \propto M \;\Rightarrow T_{\rm eff}^4 \propto \dfrac{L}{R^2} \propto M^2 \;\Rightarrow L_{\rm MS} \propto T_{\rm eff}^8 \sim T_{\rm col}^8$




*Nearby stars measured by the Hipparcos satellite*

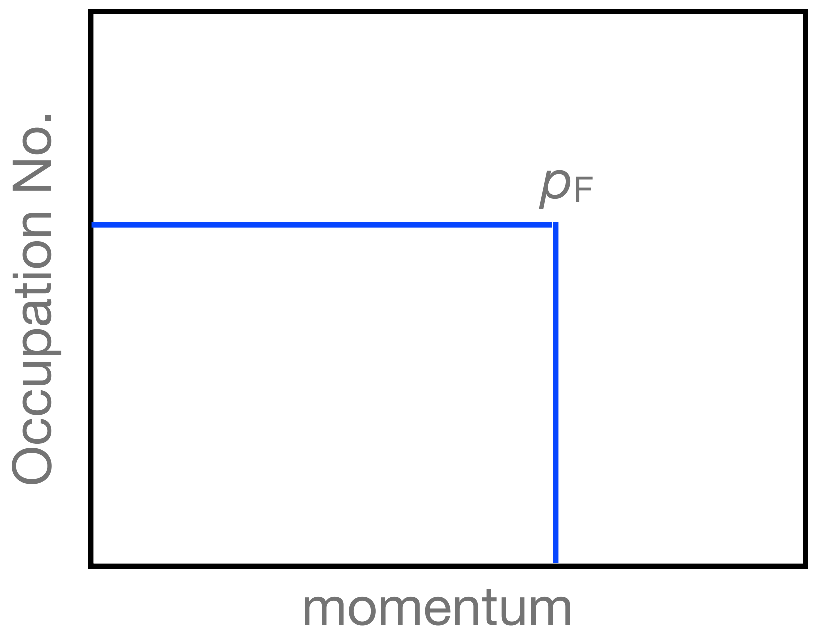
# Theoretical H-R diagram




*Stellar evolutionary tracks by Schaller et al 1992*

# Degeneracy Pressure

Momentum space occupation in cold Fermi gas




No. of particles per unit volume $n = \left(\frac{g}{h^3}\right)\frac{4\pi}{3}p_{\mathrm{F}}^3$

hence $p_{\mathrm{F}} = \left(\frac{3}{4\pi g}\right)^{1/3} hn^{1/3}$

Pressure $P \sim n \cdot v \cdot p_{\mathrm{F}} \propto v \cdot n^{4/3}$

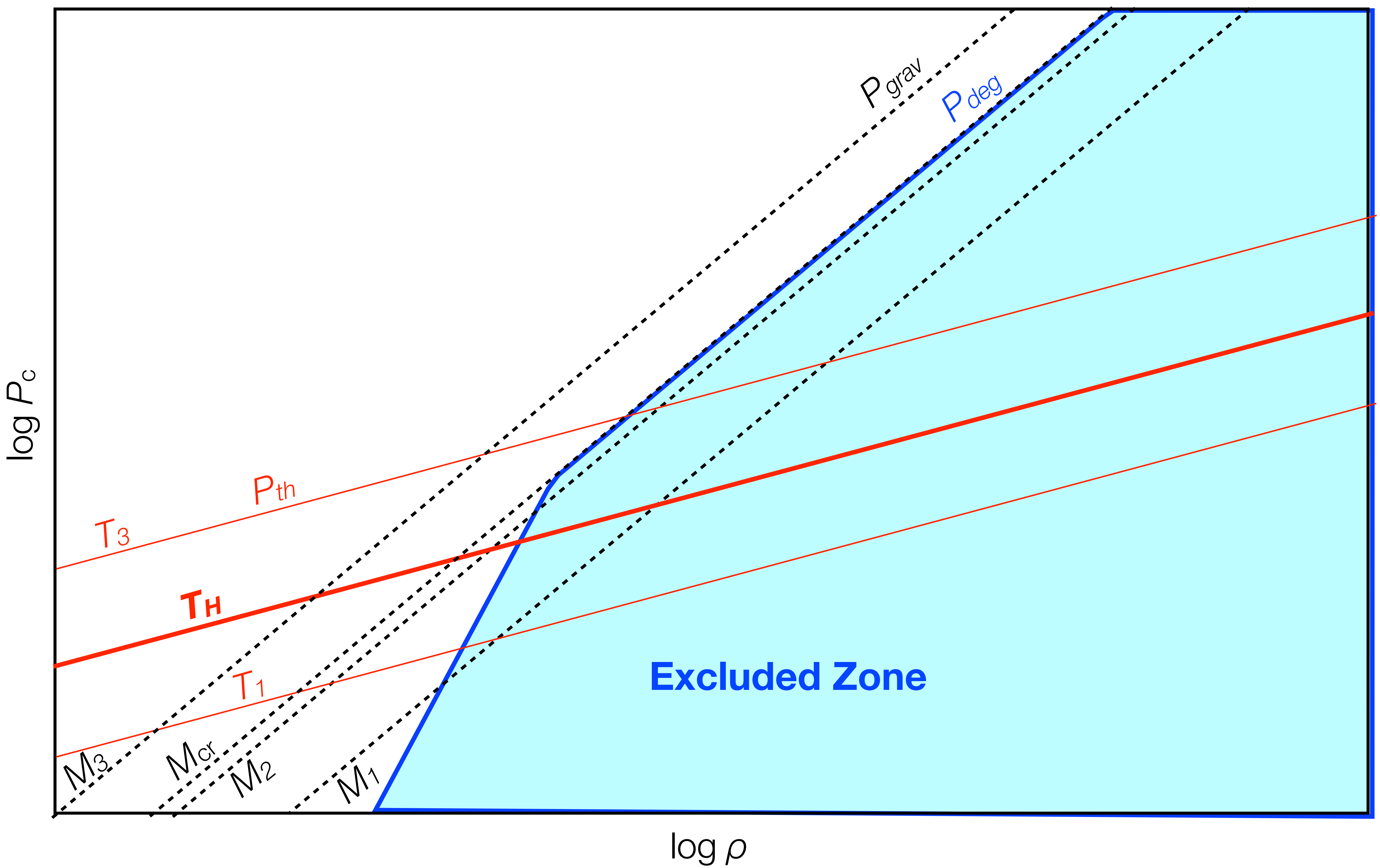
Electron degeneracy: $v = p_{\mathrm{F}}/m_{\mathrm{e}}$ (non-relativistic) and $v = c$ (relativistic)

$n_{\mathrm{e}} = \rho/(\mu_{\mathrm{e}} m_{\mathrm{p}})$ in both regimes

∴ Electron Degeneracy Pressure $P_{\mathrm{deg}} \propto \rho^{5/3}$ (non-relativistic)

$\propto \rho^{4/3}$ (relativistic)

# Stellar Equilibrium

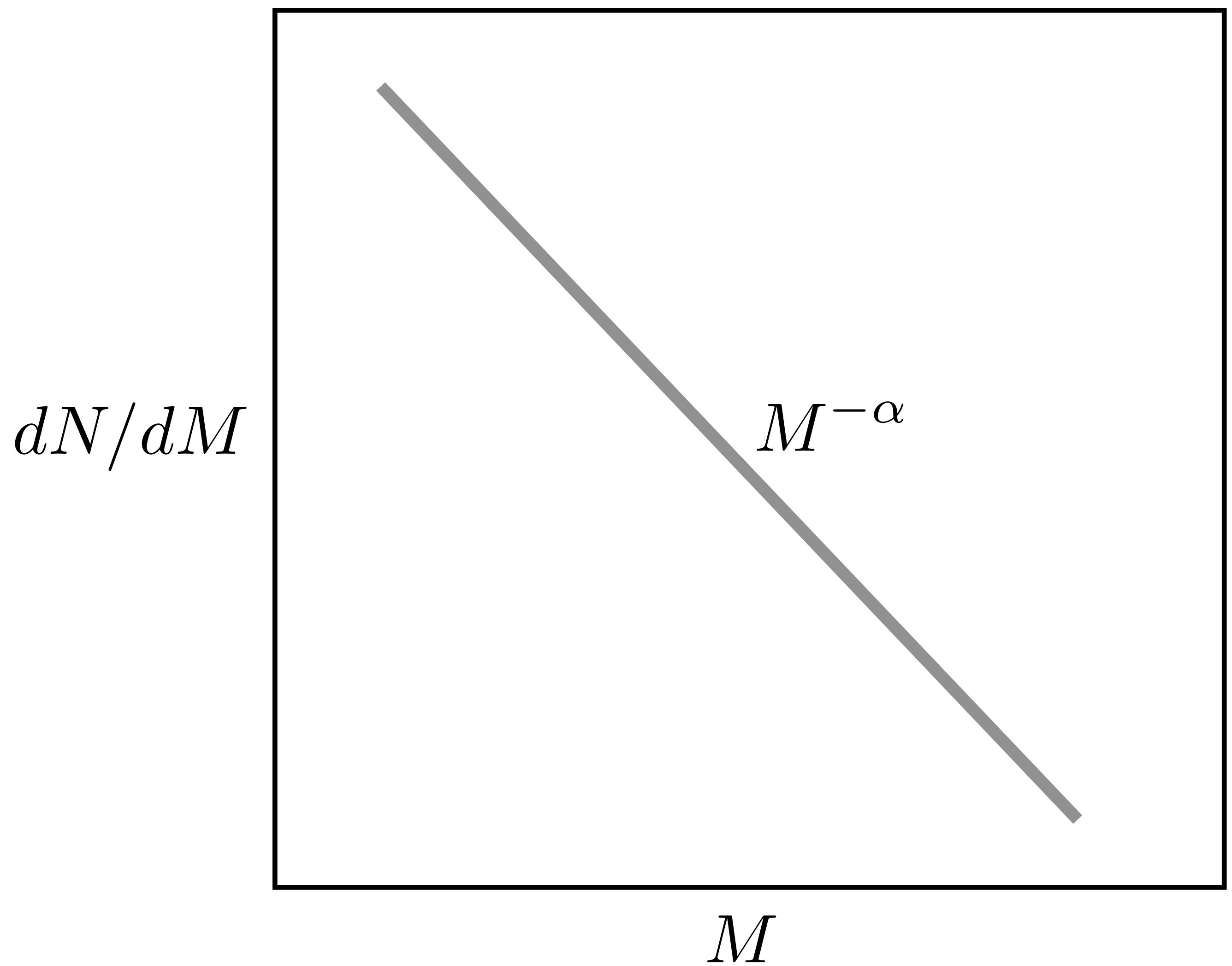


# Stellar Mass Function

The mass distribution is typically a power-law.

In the Milky Way the index $\alpha \approx 2.35$

*Salpeter (1966)*



# References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- The Physical Universe : *F.H. Shu*
- Stellar Structure and Evolution : *R. Kippenhahn and A. Weigert*
- Structure and Evolution of the Stars: *M. Schwarzcshild*
- http://www.rssd.esa.int/index.php?project=HIPPARCOS&page=HR_dia

# Compact Stars

# White Dwarfs

Configurations supported by Electron Degeneracy Pressure

$P_{\rm deg} = K_1 m_{\rm e}^{-1} (\rho/\mu_{\rm e} m_{\rm p})^{5/3}$ (non-relativistic)

$P_{\rm deg} = K_2 (\rho/\mu_{\rm e} m_{\rm p})^{4/3}$ (relativistic) : when $p_{\rm F} \gtrsim m_{\rm e} c$ ($\rho \gtrsim 10^6$ g cm$^{-3}$)

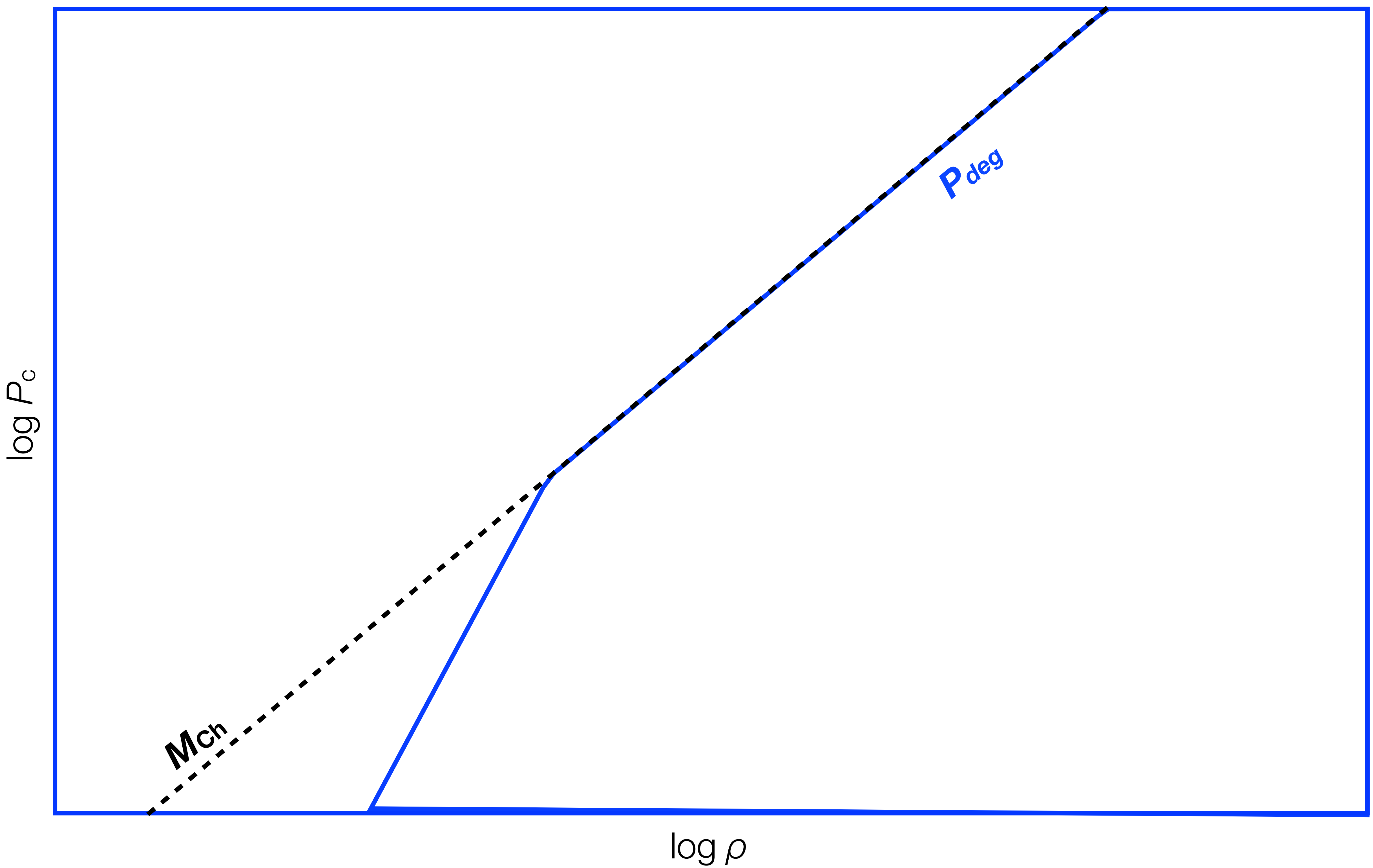
Equilibrium condition: $P_{\rm deg} \approx G M^{2/3} \rho^{4/3}$

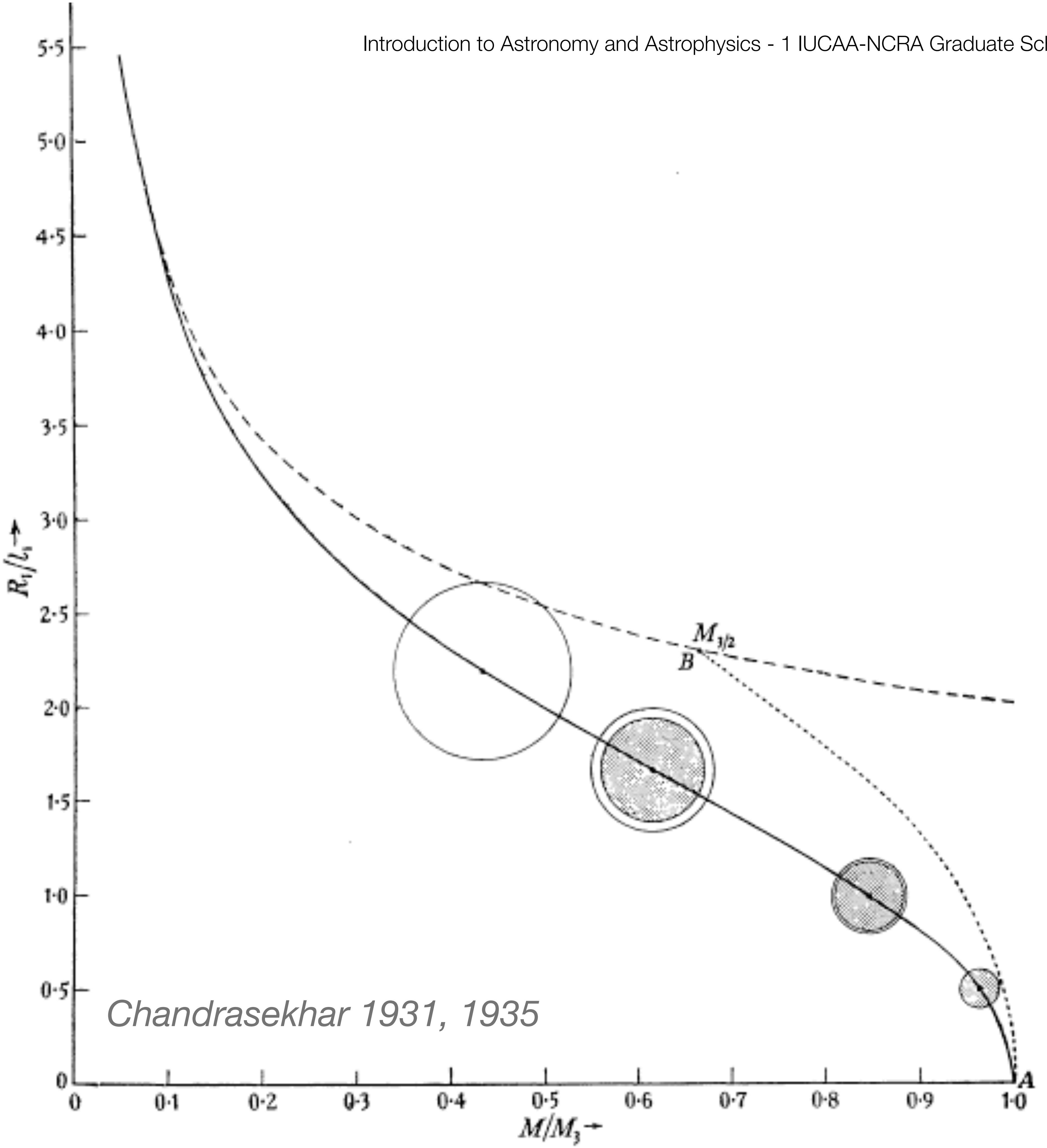
$\Rightarrow$ Non-relativistic: $R \propto m_{\rm e}^{-1} \mu_{\rm e}^{-5/3} M^{-1/3}$ ( $R$ ~ $10^4$ km for $M$ ~ 1 $M_{\rm sun}$ )

Relativistic: $M \sim \left(\frac{K_2}{G}\right)^{3/2} (\mu_{\rm e} m_{\rm p})^{-2}$ : Limiting Mass (Chandrasekhar Mass)

$$M_{\rm Ch} = 5.76 \mu_{\rm e}^{-2} \ M_\odot$$

# Limiting Mass of White Dwarf






*Chandrasekhar 1931, 1935*

# Neutron Stars

Supported by Neutron degeneracy pressure and repulsive strong interaction

TOV equation + nuclear EOS required for description

Beta equilibrium : > 90% neutrons,  < 10% protons and electrons

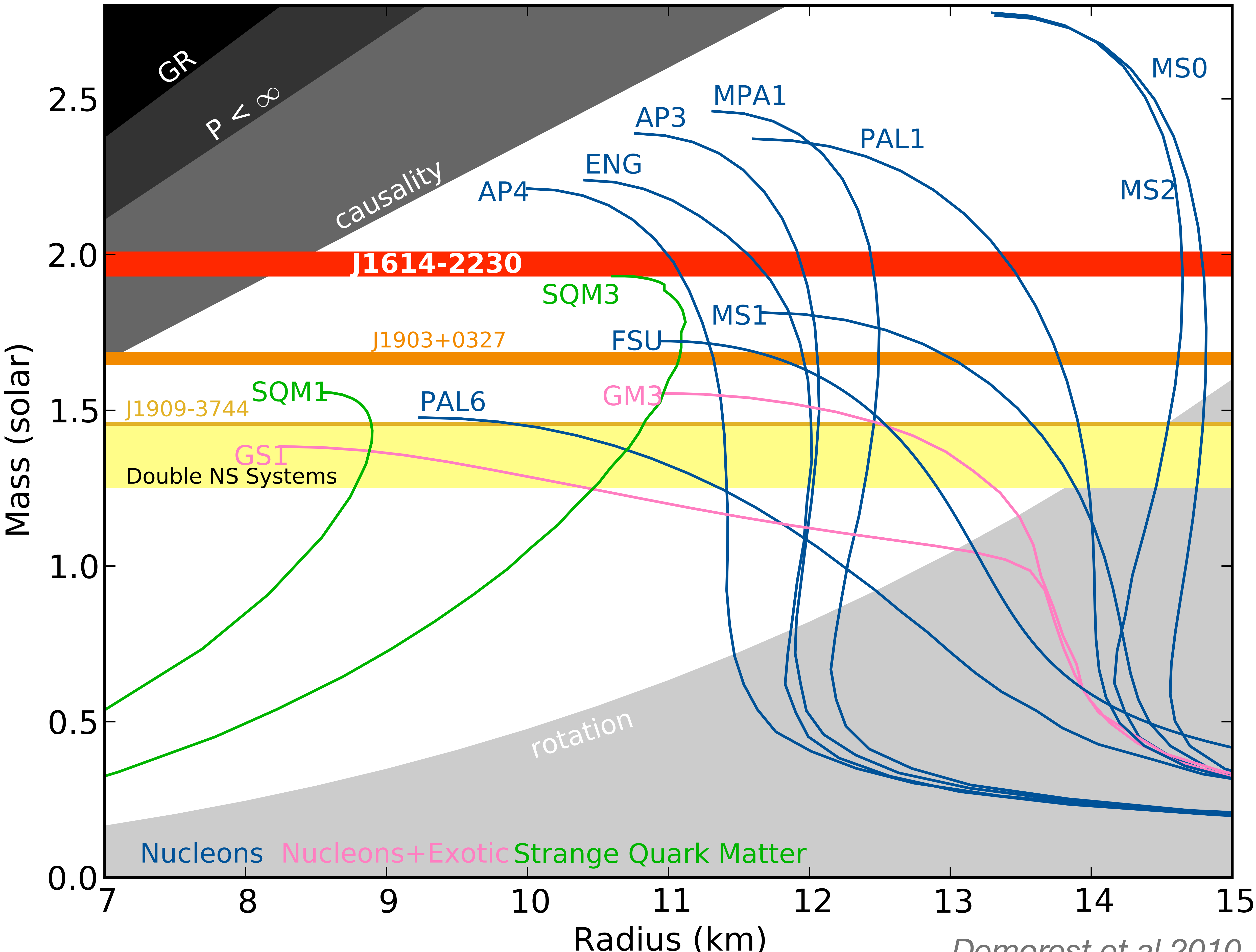
Uncertainty in the knowledge of nuclear EOS leads to uncertainty in the prediction of Mass-radius relation and limiting mass of neutron stars
*(upon exceeding the max. NS mass a Black Hole would result)*

Inter-nucleon distance ~ 1 fm ⇒ $n \sim 10^{39}$ , $\rho \sim 10^{15}$ g cm$^{-3}$

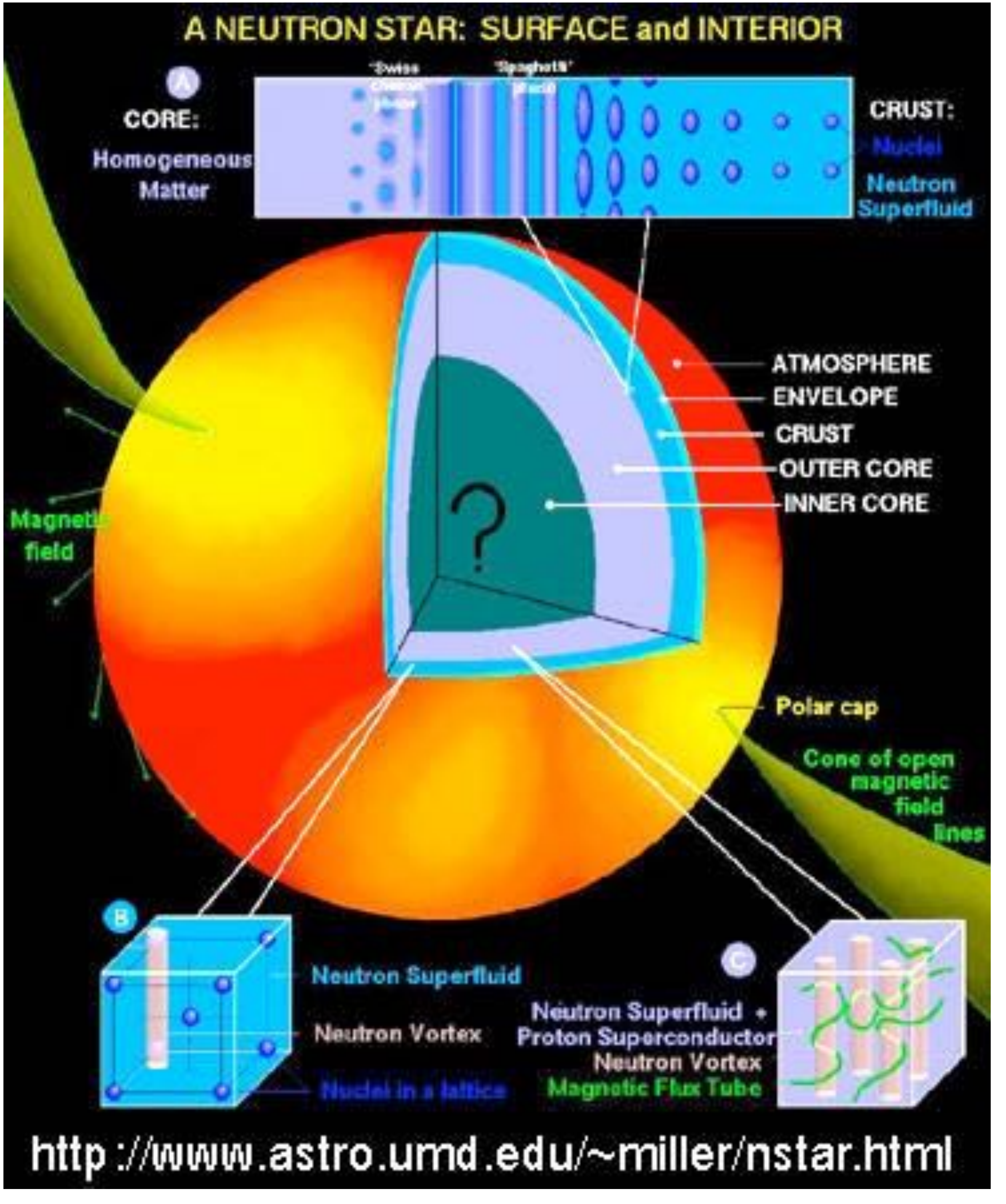
$R \sim 10$ km for $M \sim 1\ M_{\rm sun}$

Neutron stars spin fast: $P \sim$ ms - mins
and have strong magnetic field: $B_{\rm surface} \sim 10^8 - 10^{15}$ G

Exotic phenomena: *Pulsar, Magnetar activity*




*Demorest et al 2010*


A NEUTRON STAR: SURFACE and INTERIOR
A
CORE:
Homogeneous Matter
CRUST:
Nuclei
Neutron Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic field
Polar cap
Cone of open magnetic field lines
B
Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
C
Neutron Superfluid + Proton Superconductor
Neutron Vortex
Magnetic Flux Tube
http://www.astro.umd.edu/~miller/nstar.html

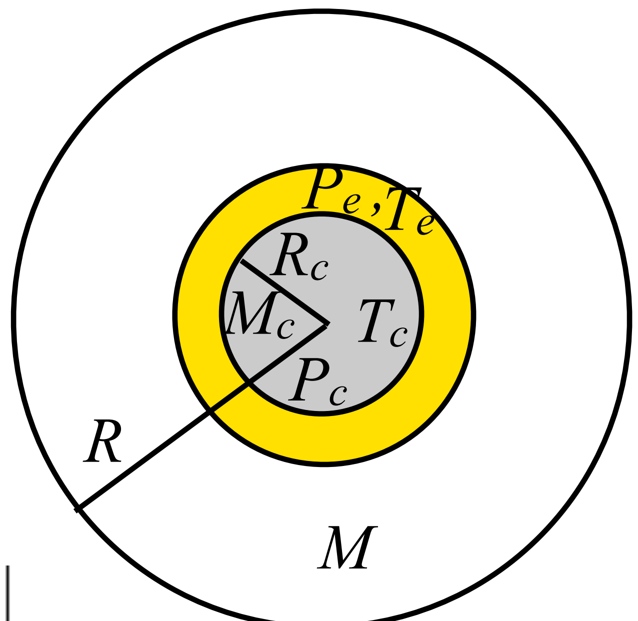
# References

- The Physical Universe : *F.H. Shu*
- Stellar Structure and Evolution : *R. Kippenhahn and A. Weigert*
- An Introduction to the Study of Stellar Structure: *S. Chandrasekhar*
- White Dwarfs, Neutron Stars and Black Holes : *S.A. Shapiro and S.L.*

# Stellar Evolution

# Schönberg-Chandrasekhar limit

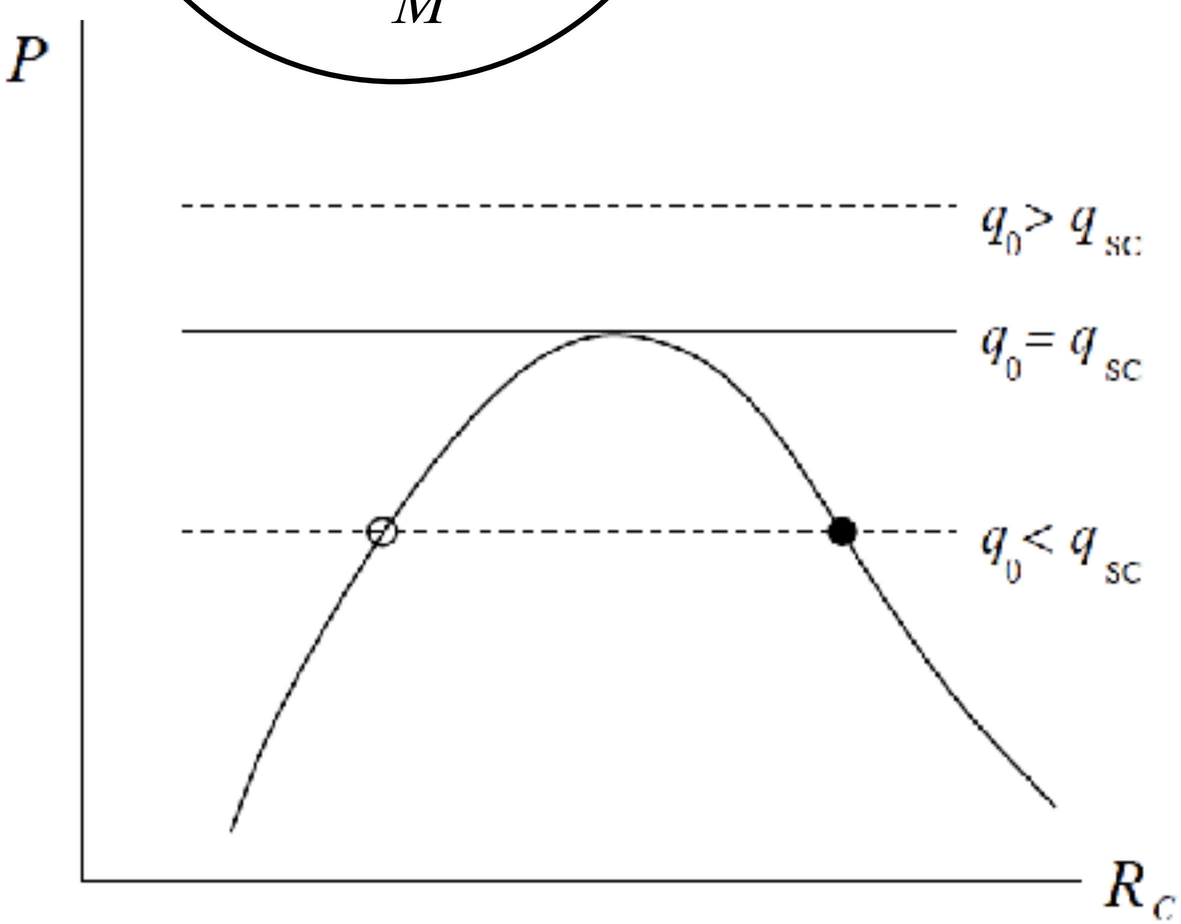
Core-envelope configuration: Inert core surrounded by burning shell




Core surface pressure $P_c = \dfrac{2E_{\rm th} + E_g}{4\pi R_c^3} = c_1 \dfrac{M_c T_c}{R_c^3} - c_2 \dfrac{M_c^2}{R_c^4}$

Envelope base pressure $P_e = c_3 \dfrac{T_e^4}{M^2}$

For mechanical and thermal balance $T_e = T_c$ and $P_e = P_c$




But $P_c$ has a maximum as a function of $R_c$

$$P_{c,\rm max} = c_4 \frac{T_c^4}{M_c^2}$$

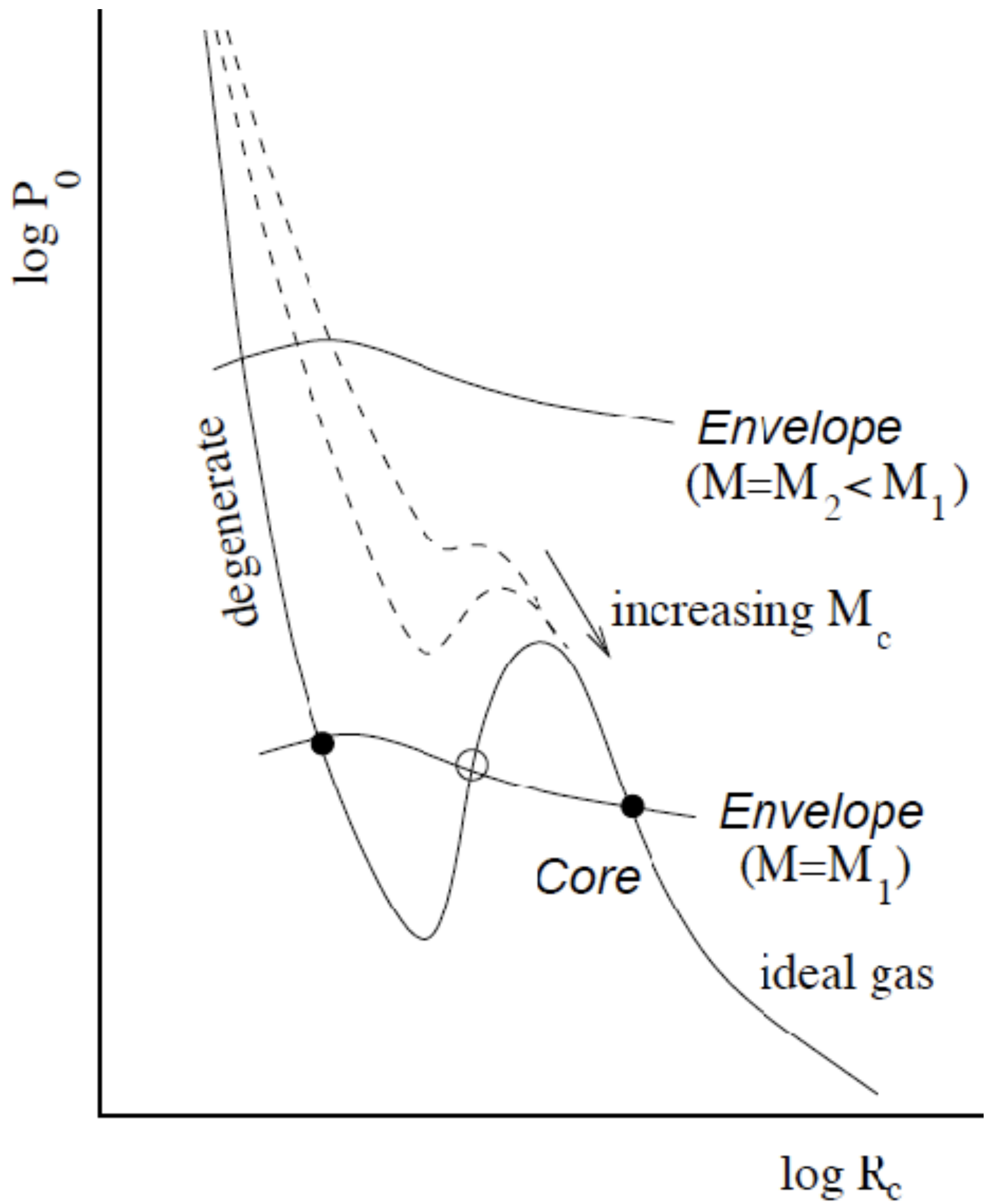
So balance is possible only if $P_e \leq P_{c,\rm max}$

i.e. $q_0 \equiv \dfrac{M_c}{M} \leq \sqrt{\dfrac{c_4}{c_3}} \equiv q_{\rm sc} \approx 0.37 \left(\dfrac{\mu_{\rm env}}{\mu_{\rm core}}\right)^2$

if core mass grows beyond this, then core collapse would occur.

⇒ *contraction until degeneracy support*

# Post Main Sequence Evolution




Low mass star (< 1.4 $M_\odot$):

gradual shrinkage of the core to degenerate configuration

He ign at $M_c$ = 0.45 $M_\odot$ : L=100 $L_\odot$

varied mass loss; *horizontal branch*

later AGB ⇒ WD+*planetary nebula*

High mass star:

sudden collapse of the core from thermal to degenerate branch

⇒ quick progress to giant

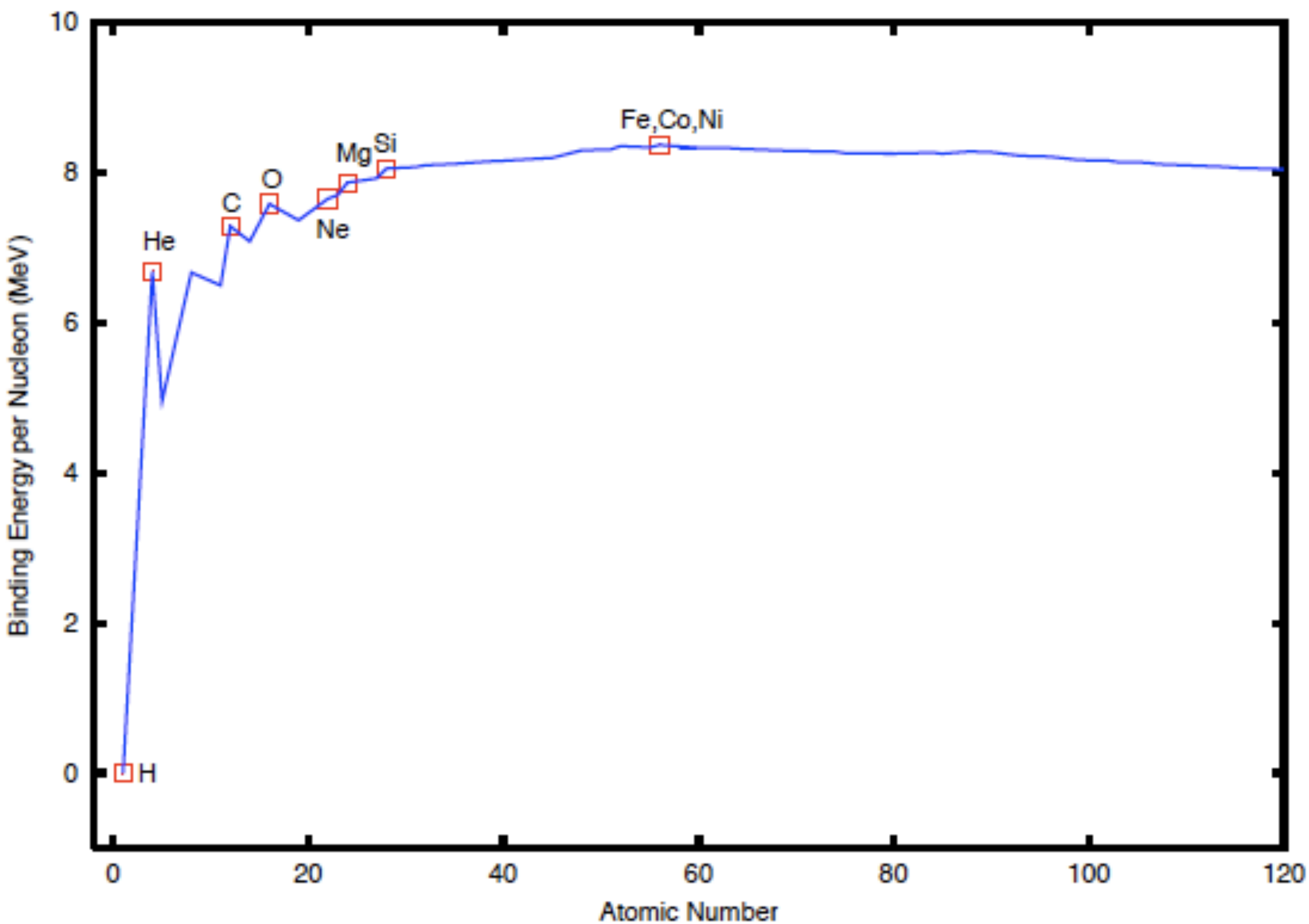
Multiple burning stages

If final degen. support at $M_c < M_{ch}$ , WD+PN will result

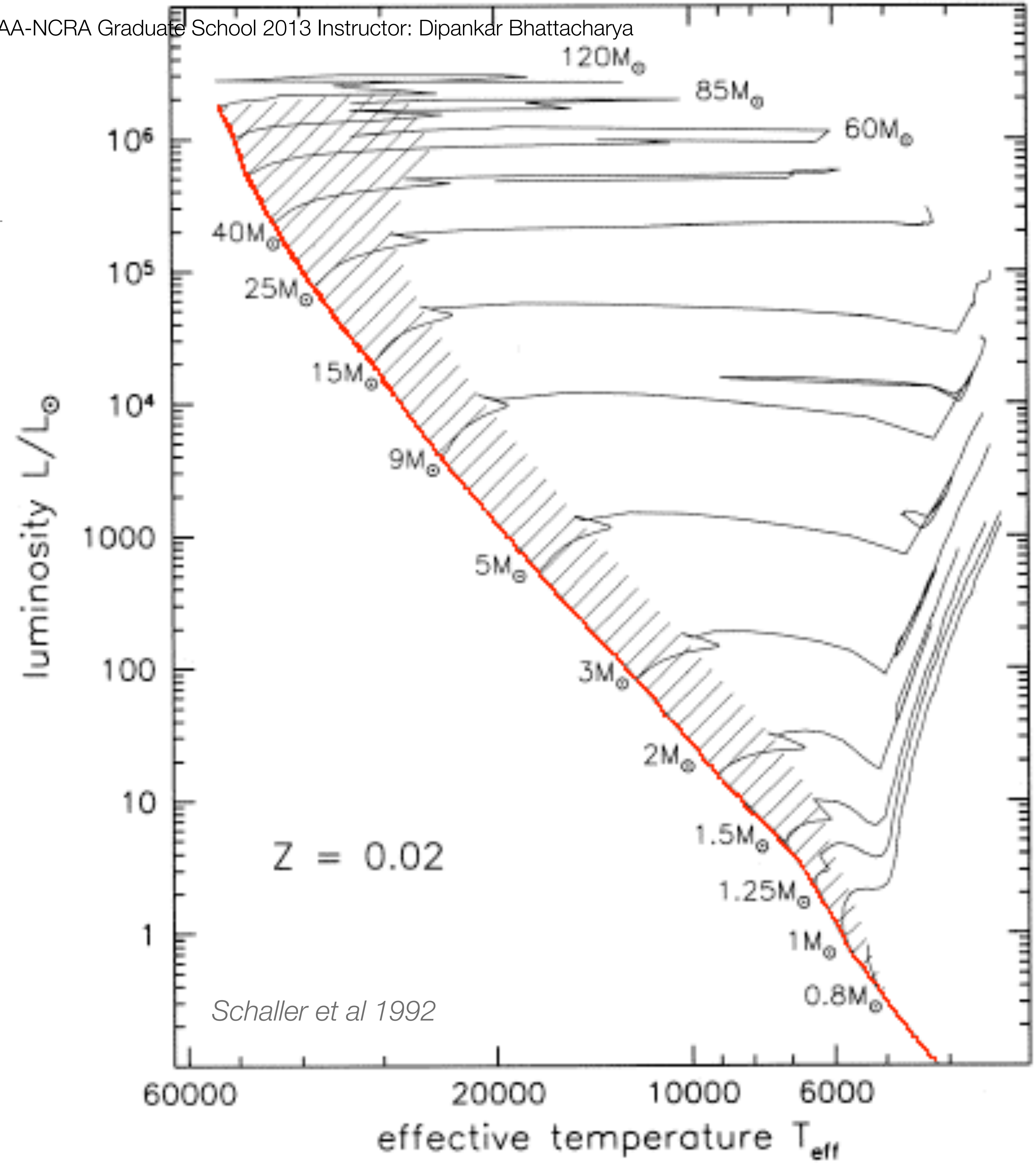
Else burning all the way to Fe core.

Once $M_{ch}$ exceeded : collapse, neutronization, supernova

# Nuclear burning stages



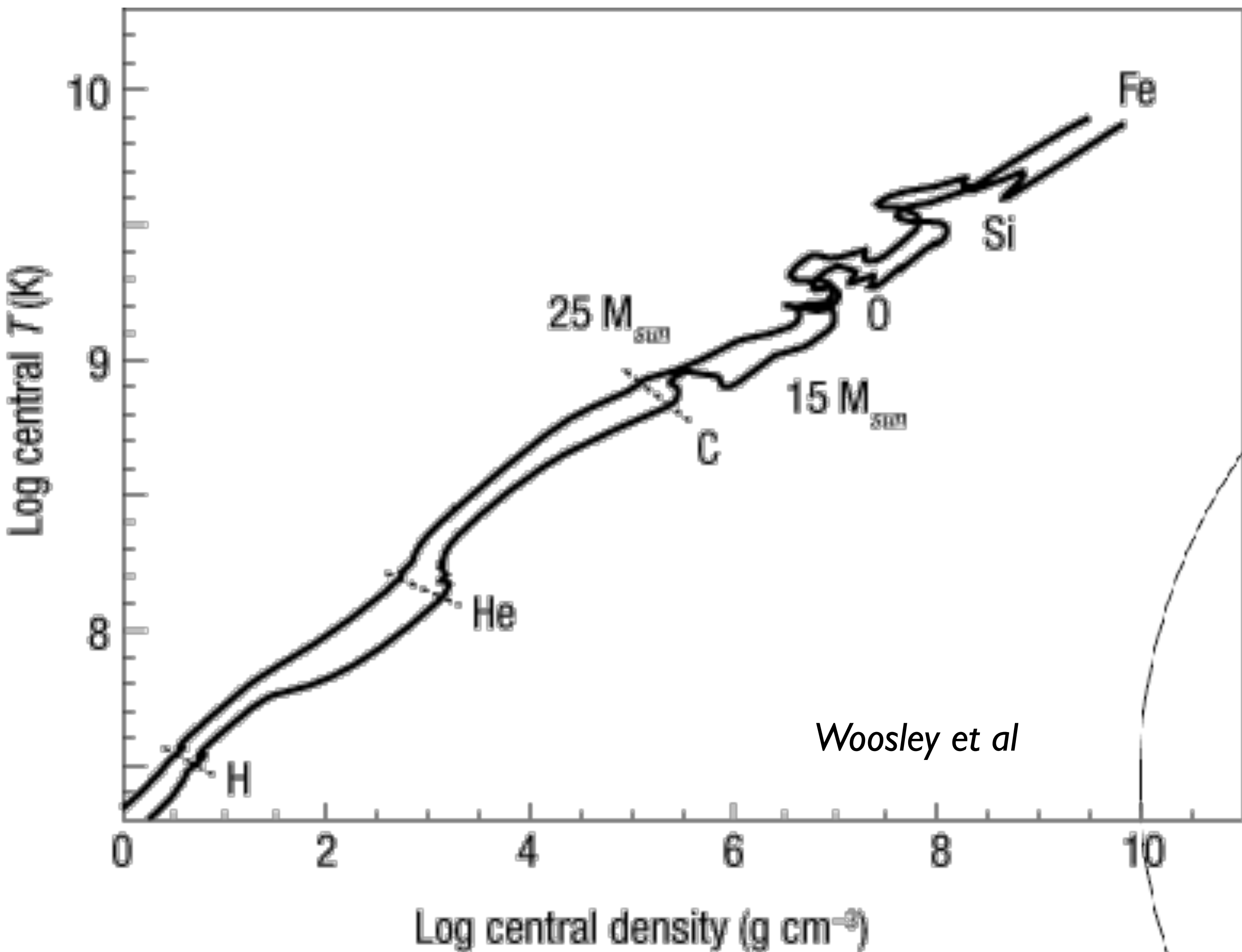
# Stellar Evolutionary Tracks

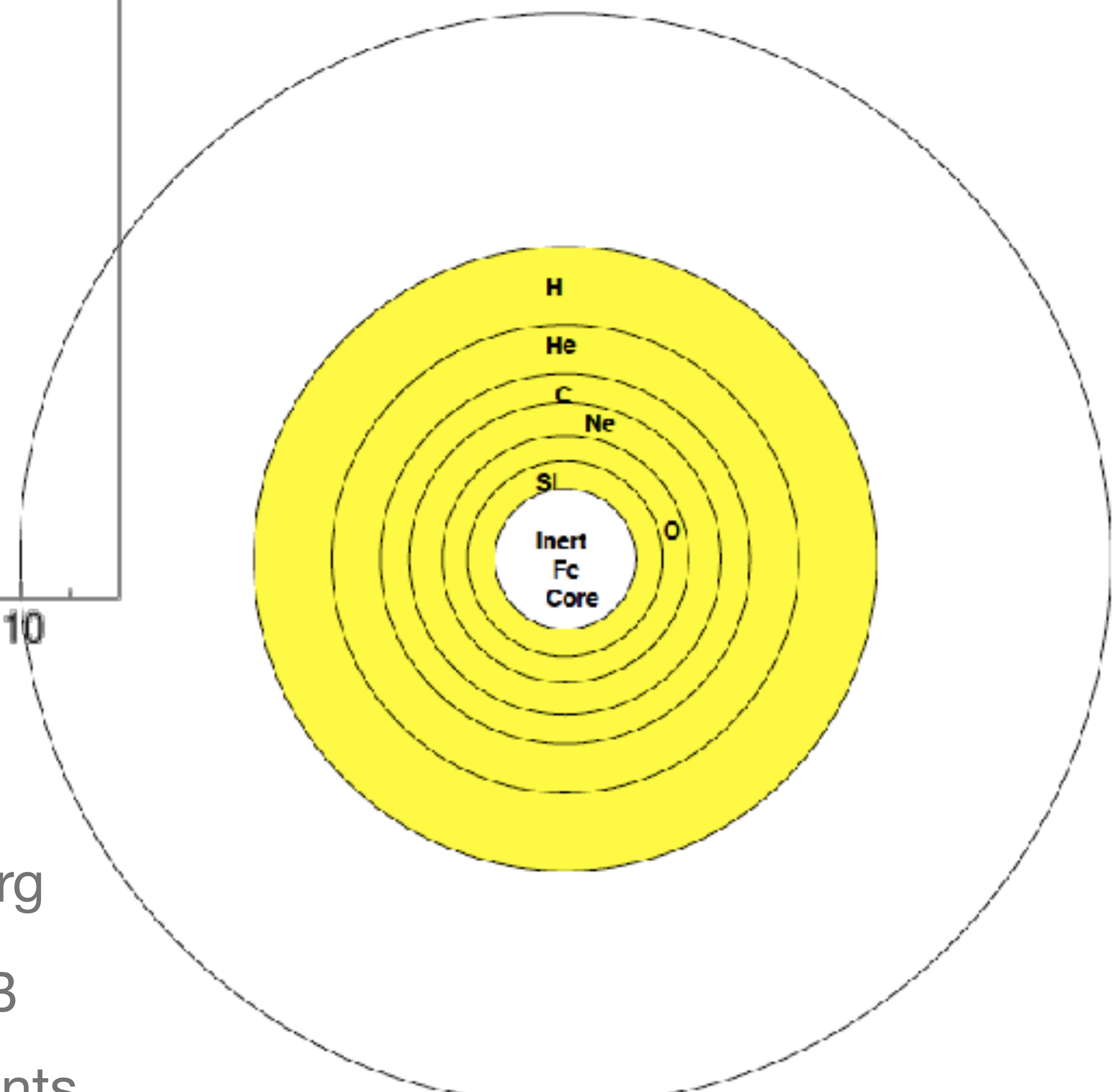



**Star Cluster Studies**

- Spectroscopic Parallax: *distance*
- Turnoff mass: *age*

# A star’s journey to a supernova




Core-collapse Supernovae:

$E_{tot}$ ~ $10^{53}$ erg; $E_{kin}$ ~ $10^{51}$ erg; $E_{rad}$ ~ $10^{49}$ erg

Massive, fast spinning stars ⇒ jets ⇒ GRB

r-process nucleosynthesis ⇒ heavy elements

# Supernovae of Type Ia

Occur due to mass transfer in double WD binary followed by complete explosion.
No core collapse

WD composition: C+O
Accretion increases WD mass ⇒ $M_{ch}$ approached ⇒ rapid contraction ⇒ heating

⇒ degenerate C-ignition ⇒ thermal runaway ⇒ explosion

$E_{tot} \sim 10^{51}$ erg ; Generates radioactive Ni which powers light curve

Standard conditions, standard appearance ⇒ Distance Indicators

# References


- Stellar Structure and Evolution : *R. Kippenhahn and A. Weigert*
- Stars: Their Birth, Life and Death : *I.S. Shklovskii*
- An Introduction to the Theory of Stellar Structure and Evolution : *D. Prialnik*
- An Invitation to Astrophysics : *T. Padmanabhan*

# Radiation

# Radiative Transfer

Radiation is modified while propagating through matter

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \quad \Rightarrow \quad \frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$\alpha_\nu$: absorption coef; $j_\nu$: emission coef

$$\Rightarrow I_\nu = S_\nu + (I_\nu^0 - S_\nu)e^{-\tau_\nu}$$

$I_\nu$ : Specific Intensity (energy/area/s/Hz/sr)

$\tau_\nu$ : Optical Depth

$S_\nu$ : Source Function

For a thermal source $S_\nu$ = Planck function (blackbody) $B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}$

$= 2\nu^2 kT/c^2$ $(h\nu \ll kT)$

$\tau_\nu \gg 1$ (optically thick) $\Rightarrow$ Blackbody radiation

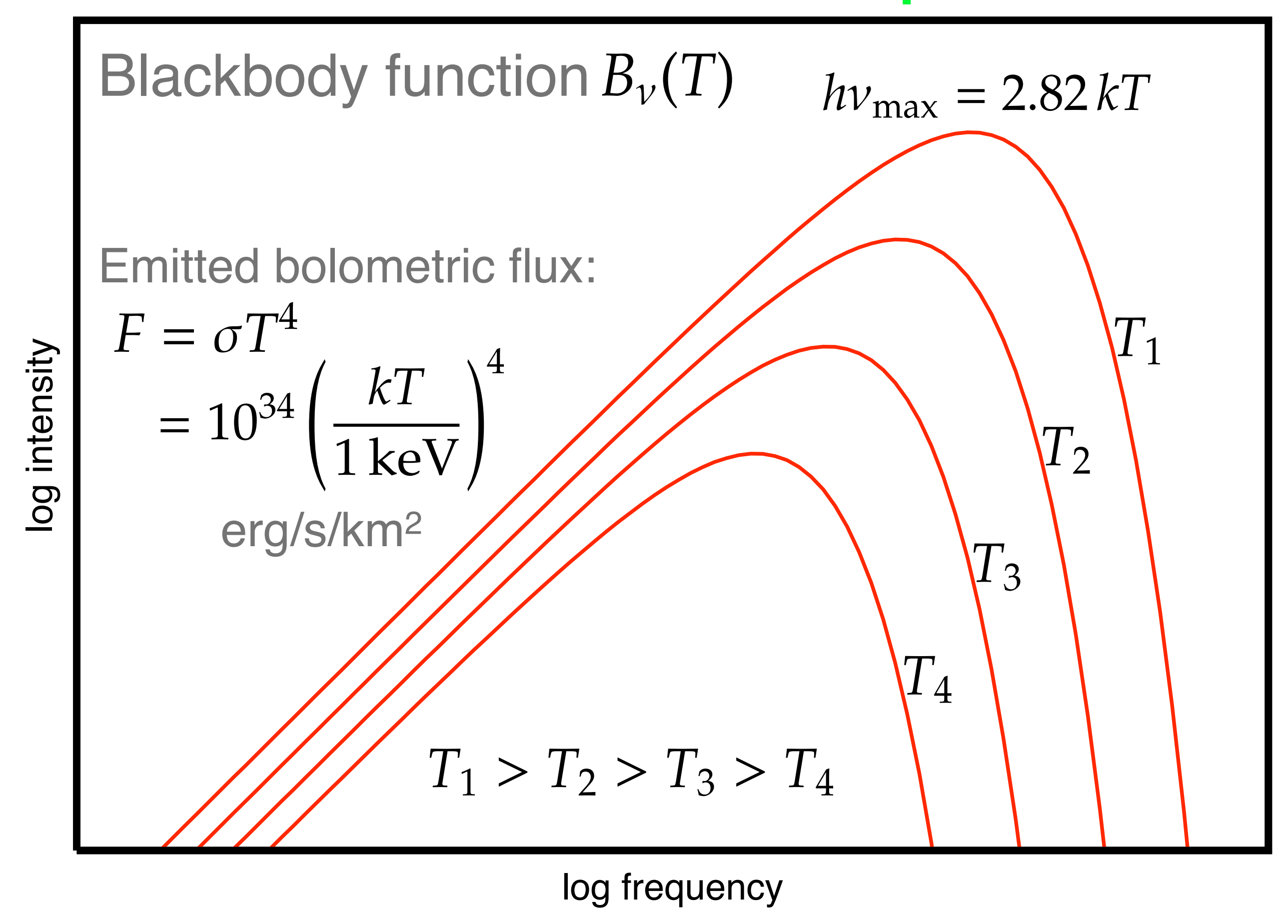
A source can be optically thick at some frequencies and optically thin at others

$\tau_\nu \ll 1$ (optically thin) $\Rightarrow I_\nu = I_\nu^0 + (S_\nu - I_\nu^0)\tau_\nu$

$T_{\rm src} > T_{\rm bg}$ : emission

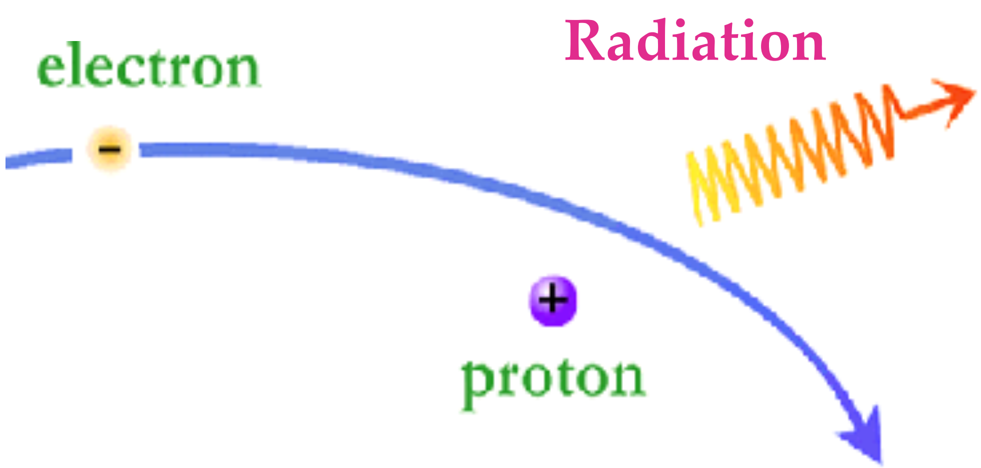
$T_{\rm src} < T_{\rm bg}$ : absorption

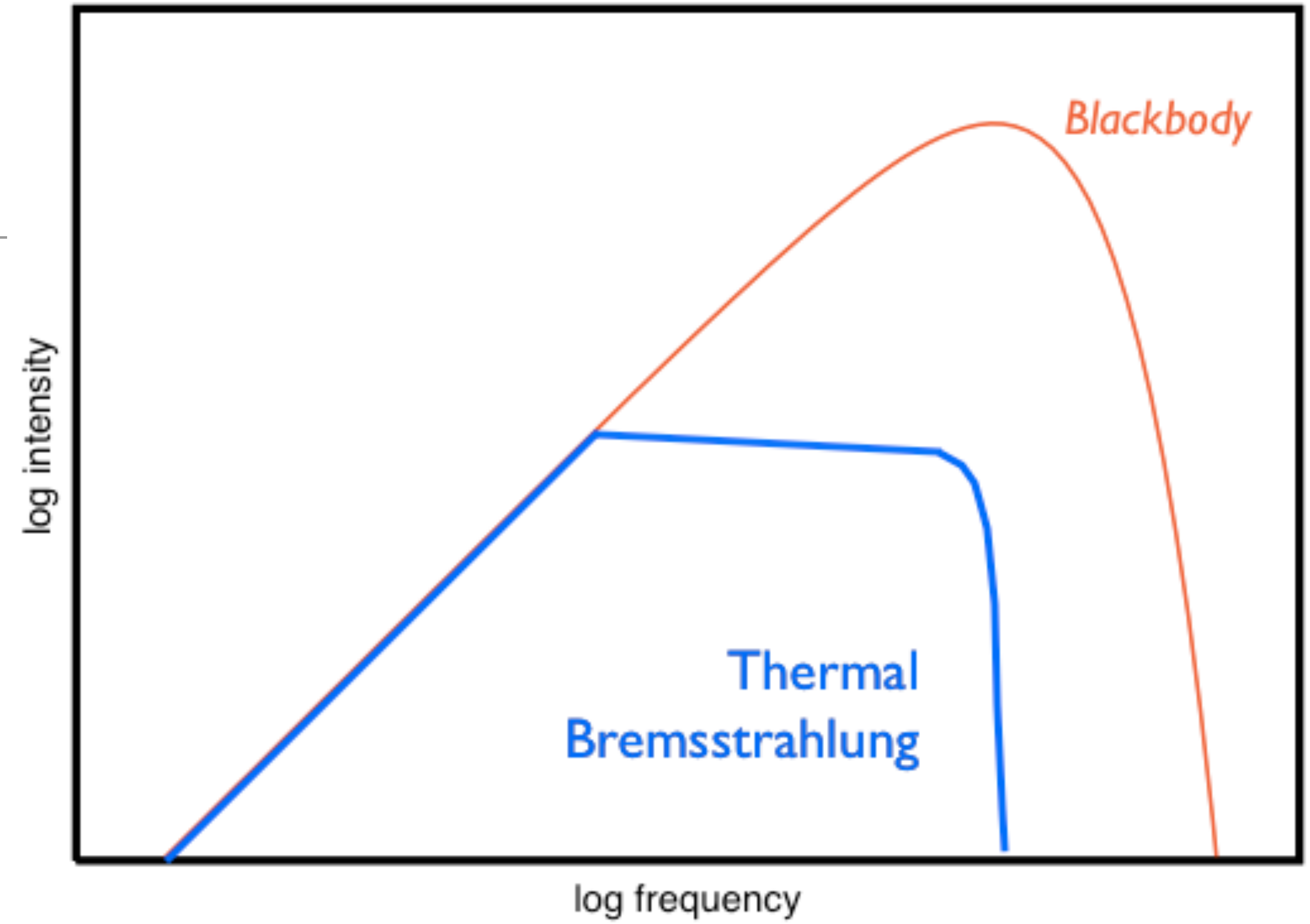
*For non-thermal distribution of particle energies,* $S_\nu \neq B_\nu$



# Bremsstrahlung

(free-free process)




Emission coefficient :

$$\epsilon_\nu^{\rm ff} = 6.8 \times 10^{-38} Z^2 n_{\rm e} n_{\rm i} T^{-1/2} {\rm e}^{-h\nu/kT} \bar{g}_{\rm ff} \text{ erg s}^{-1} \text{ Hz}^{-1} \text{ cm}^{-3}$$

all $n$ values in cm$^{-3}$

Bolometric: $$\epsilon^{\rm ff} = 1.4 \times 10^{-27} Z^2 n_{\rm e} n_{\rm i} T^{1/2} \bar{g}_{\rm ff} \text{ erg s}^{-1} \text{ cm}^{-3}$$

Free-free absorption coefficient:

$$\alpha_\nu^{\rm ff} = 3.7 \times 10^{8} Z^2 n_{\rm e} n_{\rm i} T^{-1/2} \nu^{-3} \left(1 - {\rm e}^{-h\nu/kT}\right) \bar{g}_{\rm ff} \text{ cm}^{-1}$$

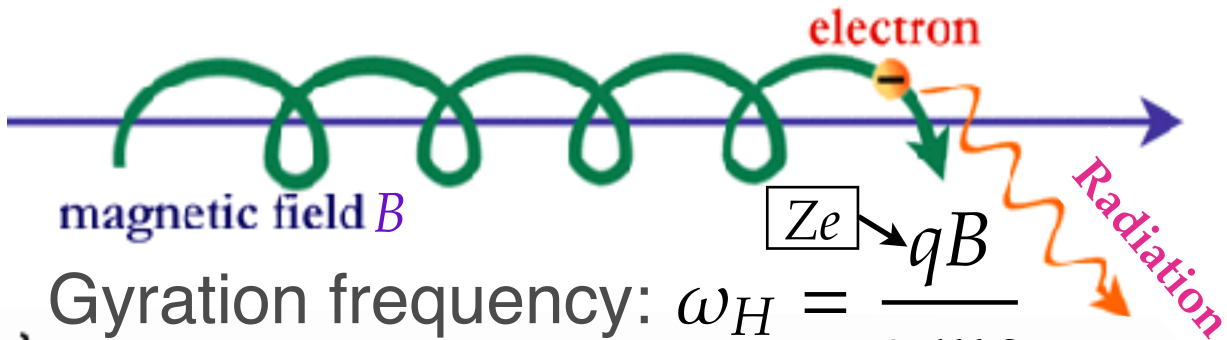
$$\approx 4.5 \times 10^{-25} Z^2 n_{\rm e} n_{\rm i} (kT)_{\rm keV}^{-3/2} \nu_{\rm MHz}^{-2} \bar{g}_{\rm ff} \text{ cm}^{-1}$$

Compare Thomson:

$$\alpha_{\rm T} = n_{\rm e} \sigma_{\rm T}$$
$$= 6.65 \times 10^{-25} n_{\rm e} \text{ cm}^{-1}$$

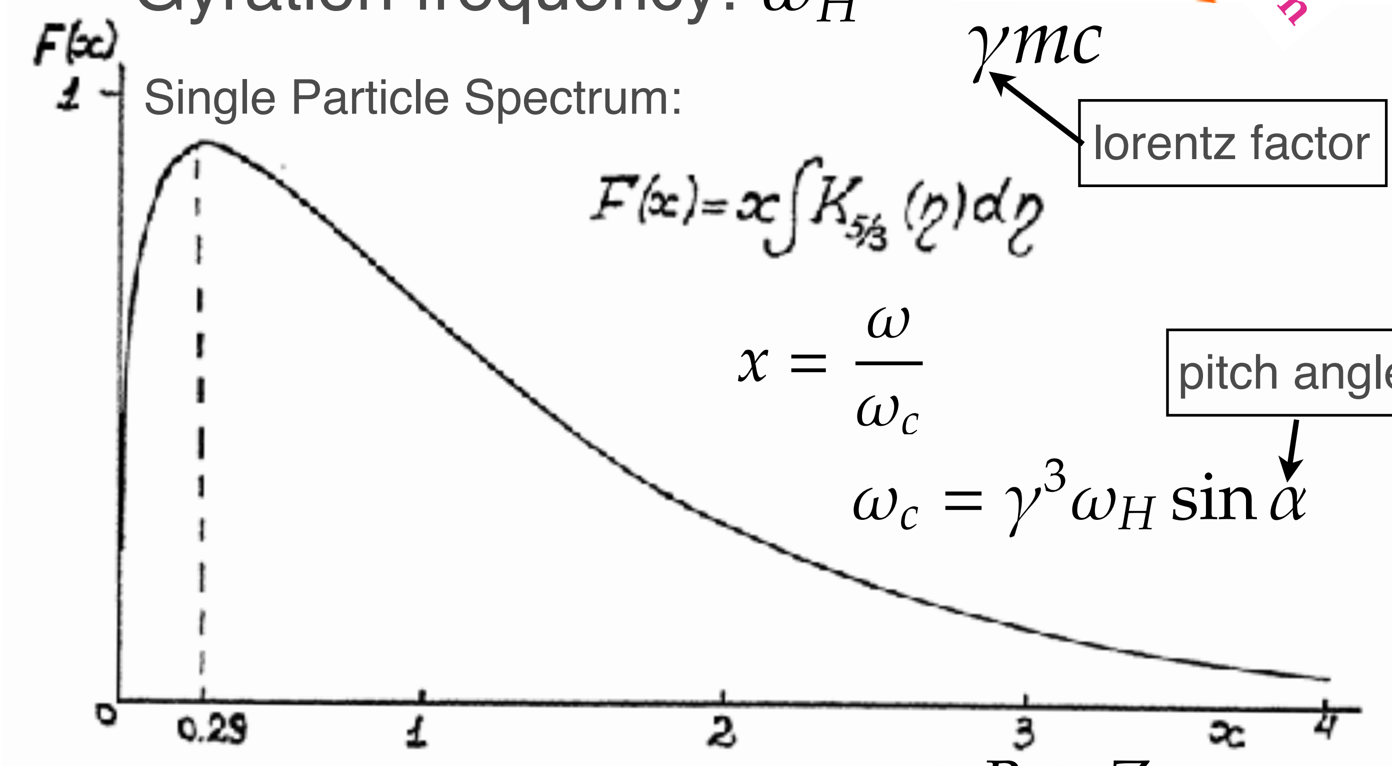
# Synchrotron

(relativistically moving charged particle in a magnetic field)




Gyration frequency: $\omega_H = \frac{qB}{\gamma mc}$

lorentz factor




Single Particle Spectrum:

$F(x) = x\int K_{5/3}(\eta)d\eta$

$x = \frac{\omega}{\omega_c}$

$\omega_c = \gamma^3 \omega_H \sin\alpha$

pitch angle

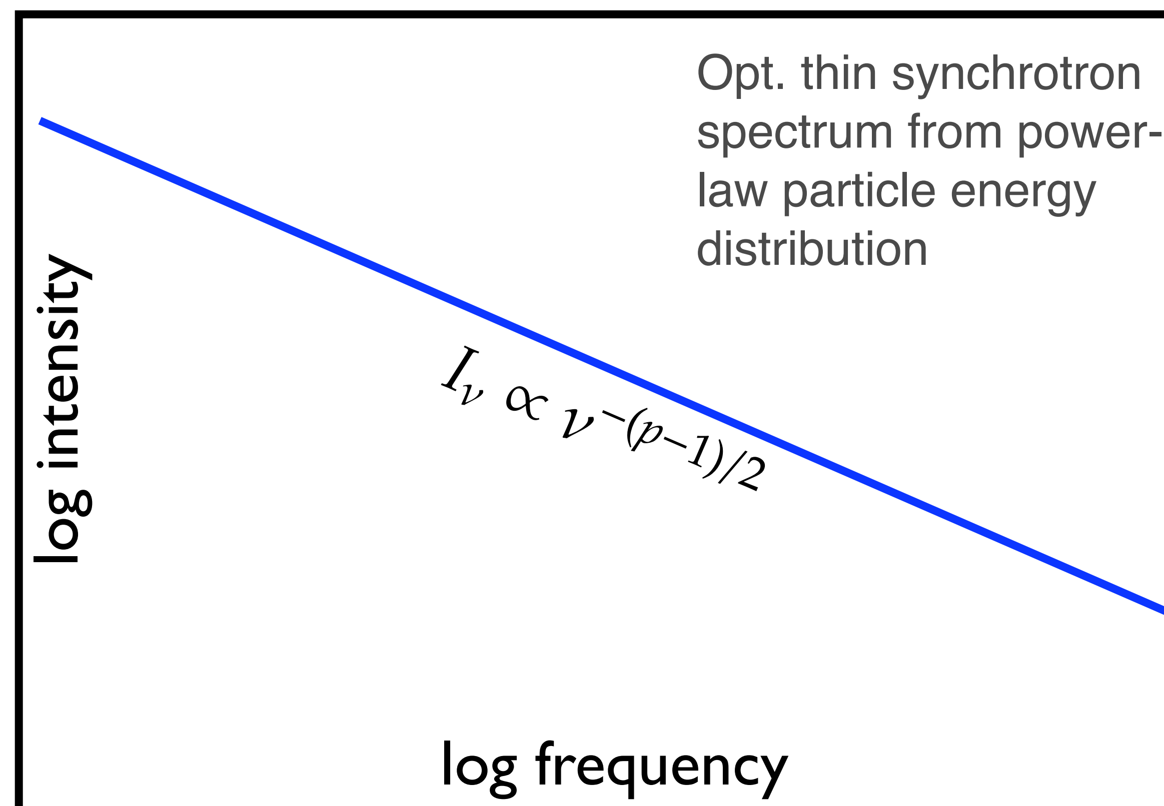
$$\nu_{\rm peak} = 0.29\frac{\omega_c}{2\pi} \approx 0.81\gamma^2\left(\frac{B}{1\,\rm G}\right)\frac{Zm_e}{m}\ \text{MHz}$$

$$\text{Power} = \frac{4}{3}\sigma_T c\beta^2\gamma^2 U_B\left(\frac{Z^2 m_e}{m}\right)^2$$

$$= 2.0\times10^{-14}\gamma^2\left(\frac{B}{1\,\rm G}\right)^2\left(\frac{Z^2 m_e}{m}\right)^2 \text{ erg / s per particle}$$

Particle acceleration process often generates *non-thermal*, power-law energy distribution of the relativistic charged particles: $N(\gamma) \propto \gamma^{-p}$

The radiation spectrum generated by such a distribution is also a power-law:

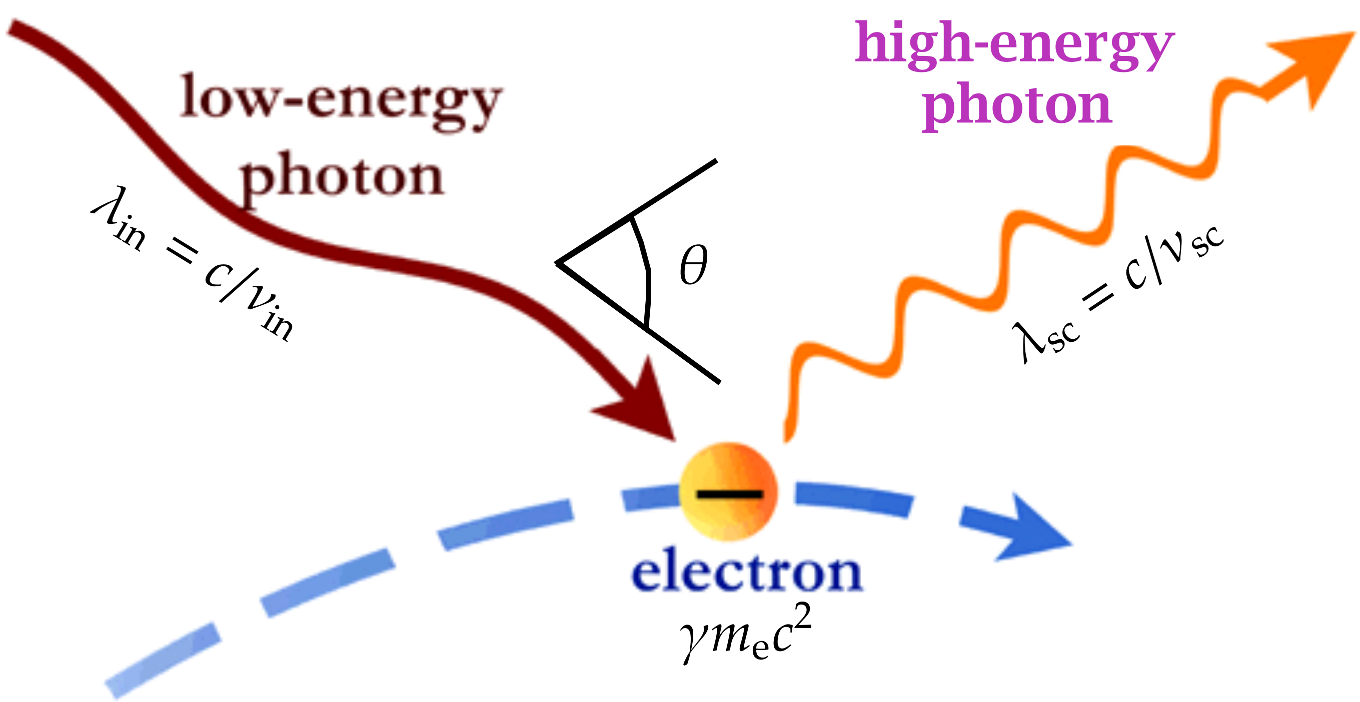



$j_\nu \propto \nu^{-(p-1)/2}$

$\alpha_\nu \propto \nu^{-(p+4)/2}$

$S_\nu \propto \nu^{5/2}$

# Compton scattering




In the frame where the electron is initially at rest,

$$\lambda_{sc} - \lambda_{in} = \frac{h}{m_e c}(1 - \cos\theta)$$

and cross section: *Klein-Nishina*

$$\sigma_{KN} \approx \sigma_T \qquad (h\nu_{in} \ll m_e c^2)$$
$$\propto \nu_{in}^{-1} \qquad (h\nu_{in} \gg m_e c^2)$$

In the observer's frame, where the electron is moving with a lorentz factor $\gamma$,

$$\nu_{sc} \approx \gamma^2 \nu_{in} \qquad \text{for } h\nu_{in} \ll m_e c^2/\gamma$$

(inverse compton scattering)

Inverse compton power emitted per electron: $\frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_{ph}$ ; $U_{ph}$ = photon energy density

Non-thermal comptonization ⇒ spectral shape akin to synchrotron process

Thermal comptonization ⇒ number conserving photon diffusion in energy space
⇒ power-law, modified blackbody or Wien spectrum
(for different limiting cases of opt. depth and y-parameter)

***Compton y parameter = (av. no. of scatterings) x (mean fractional energy change per scattering)***

# References


- Radiative Processes in Astrophysics : *G.B. Rybicki and A.P. Lightman*
- High Energy Astrophysics : *M. Longair*
- The Physics of Astrophysics vol. I: Radiation : *F.H. Shu*
- Theoretical Astrophysics vol. 1 : *T. Padmanabhan*

# Diffuse Matter

# Diffuse matter between stars: the ISM

Interstellar matter exists in a number of phases, of different temperatures and densities. Average density of Interstellar medium is ~ 1 atom / cm$^3$

At such low densities heat transfer between different phases is very slow.
So phases at multiple temperatures coexist at pressure equilibrium.

Cooling: free-free/free-bound continuum, atomic and molecular lines, dust radn.
Heating: Cosmic Rays, Supernova Explosions, Stellar Winds, Photoionization

Ionization fraction $x$: $\frac{x^2}{1-x} = \frac{2g_\mathrm{i}}{g_0 n}\left(\frac{2\pi m_\mathrm{e} kT}{h^2}\right)^{3/2} \mathrm{e}^{-\chi/kT}$ (*Saha equation*)

⇒ Hydrogen is fully ionized at T > 10$^4$ K

Ionization states are denoted by roman numeral: I: neutral, II: singly ionized.......

Gas around hot stars are photoionized by the stellar UV photons.
Ionization balance: *Recombination rate* = *Rate of supply of ionizing photons*

Strömgren sphere: $\frac{4\pi}{3} R^3 n_\mathrm{e} n_\mathrm{i} \alpha = \dot{N}_\mathrm{UV} \quad \Rightarrow \quad R = \left(\frac{3\dot{N}_\mathrm{UV}}{4\pi n_\mathrm{e}^2 \alpha}\right)^{1/3}$ (singly ionized: HII region)

recombination coef ($\alpha$)

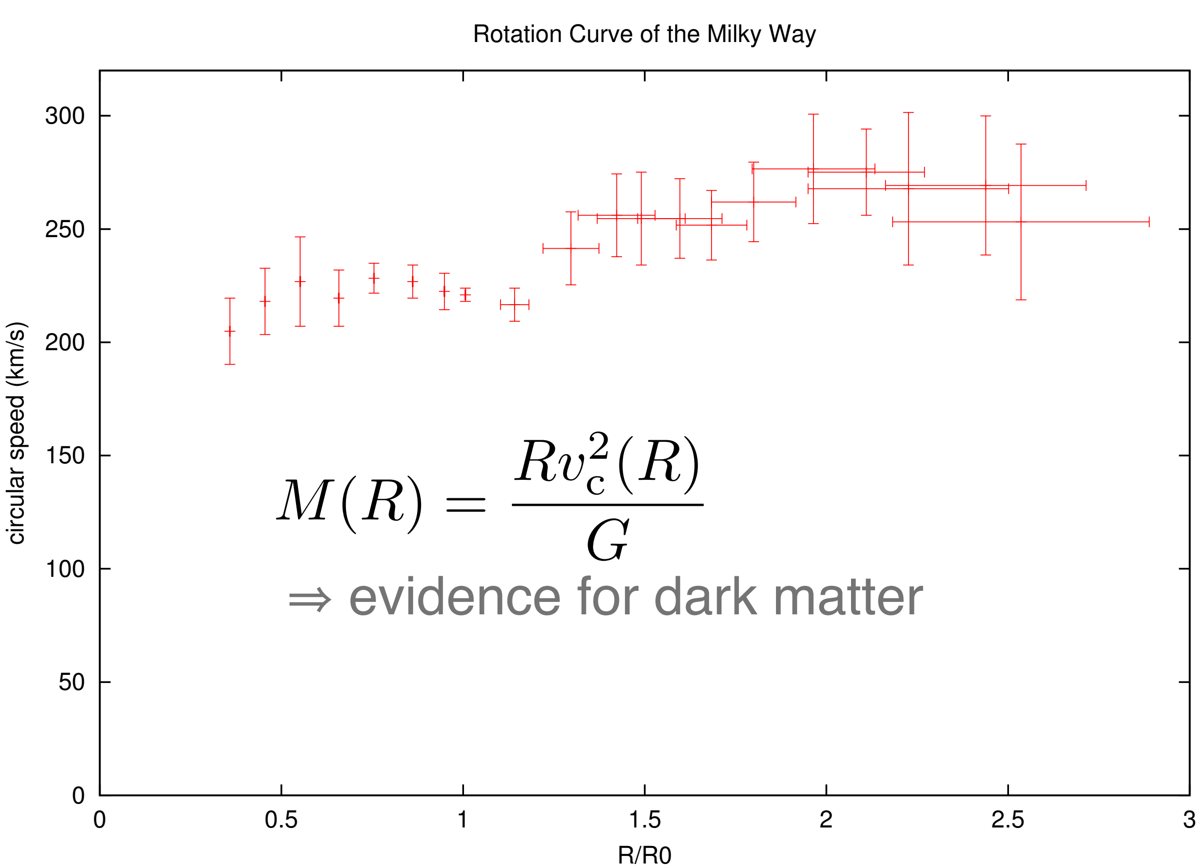
# ISM phases and constituents

- Coronal Gas: T ~ $10^6$ K
- Warm Ionized Medium: T ~ 8000 K
- Warm Neutral Medium: T ~ 6000 K
- Cold Neutral Medium: T ~ 80 K (HI clouds)
- Molecular Clouds: T < 20 K

Cosmic rays, diffuse starlight, magnetic field: ~ 1eV/cm$^3$ each

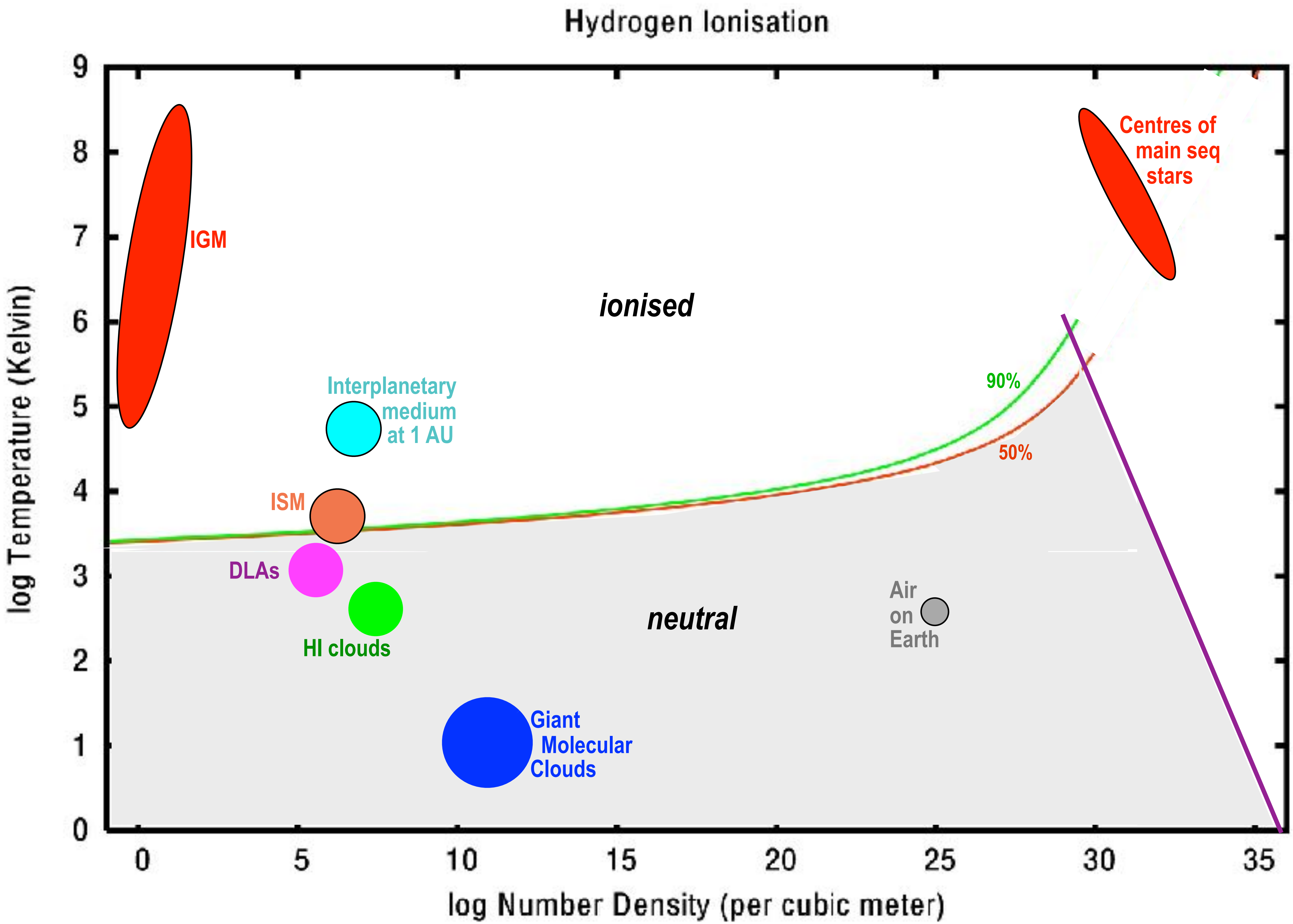
Dust: solid particles - graphite, silicates, PAHs etc.

Distributed HI gas produces 1420 MHz (21-cm) hyperfine transition line.
⇒ vital probe of density, temperature, kinematics (*e.g. rotation curve*)




Molecular regions can be studied through mm-wave rotational transitions, e.g. of CO

Dust causes extinction and reddening, re-radiates energy in Infrared, polarizes starlight, provides catalysis for molecule formation, shields molecular clouds from radiation damage, depletes the diffuse gas of some elements.


Hydrogen Ionisation
log Temperature (Kelvin)
log Number Density (per cubic meter)
IGM
Centres of main seq stars
ionised
neutral
Interplanetary medium at 1 AU
ISM
DLAs
HI clouds
Giant Molecular Clouds
Air on Earth
90%
50%
0 1 2 3 4 5 6 7 8 9
0 5 10 15 20 25 30 35

# Star Formation

Stars form by gravitational collapse and fragmentation of dense molecular clouds

Gravitational Instability occurs at masses larger than the Jeans' scale $M_J \sim \rho_0 L_J^3$

where $\frac{GM_J}{L_J}\mu m_p = kT_0 \Rightarrow L_J = \left[\frac{kT_0}{G\mu m_p \rho_0}\right]^{1/2} \Rightarrow M_J = \left[\frac{kT_0}{G\mu m_p}\right]^{3/2}\frac{1}{\rho_0^{1/2}}$

Collapse can proceed only in presence of cooling. Hence star formation rate is strongly dependent on cooling. Cooling is provided by atomic and molecular transitions. More molecules ⇒ faster cooling.

Dust aids the formation and survival of molecules.
Formation of dust needs heavy elements

In early epochs, star formation was slow; fewer, very massive stars formed
With enrichment, star formation rate (SFR) increased, many small stars produced

Infrared emission from hot dust is tracer of star formation activity
Ultra-Luminous Infra Red Galaxies (ULIRGs): example of high SFR

High SFR → High SN rate → More CR → stronger Sychrotron emission (radio)
⇒ *Radio - FIR* correlation

# Free Electrons

Ionization provides free electrons in the ISM: $\langle n_e \rangle \sim 0.03\,\mathrm{cm}^{-3}$

Propagation through this plasma causes Dispersion of e.m. radiation, measurable at radio wavelengths. Can be used to infer distances of, *e.g.* pulsars.
Plasma frequency of the ISM $\omega_p \sim 6\,\mathrm{kHz}$

Magnetic Field permeates the Interstellar Medium.
Polarized radio waves undergo Faraday Rotation while propagating through the magnetized plasma. ⇒ *key probe of distributed magnetic field*

Typical interstellar field strength: $\sim 1\,\mu\mathrm{G}$
Cyclotron resonance: $\omega_c \sim 20\,\mathrm{Hz}$

Dispersion relation of circularly polarized eigenmodes:

$$k_{r,l} = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{2\omega^2}\left(1 \mp \frac{\omega_c}{\omega}\right)\right] \qquad \text{for } \omega \gg \omega_p, \omega_c$$

Dispersion Measure:

$$DM = \int_0^L n_e\, ds$$

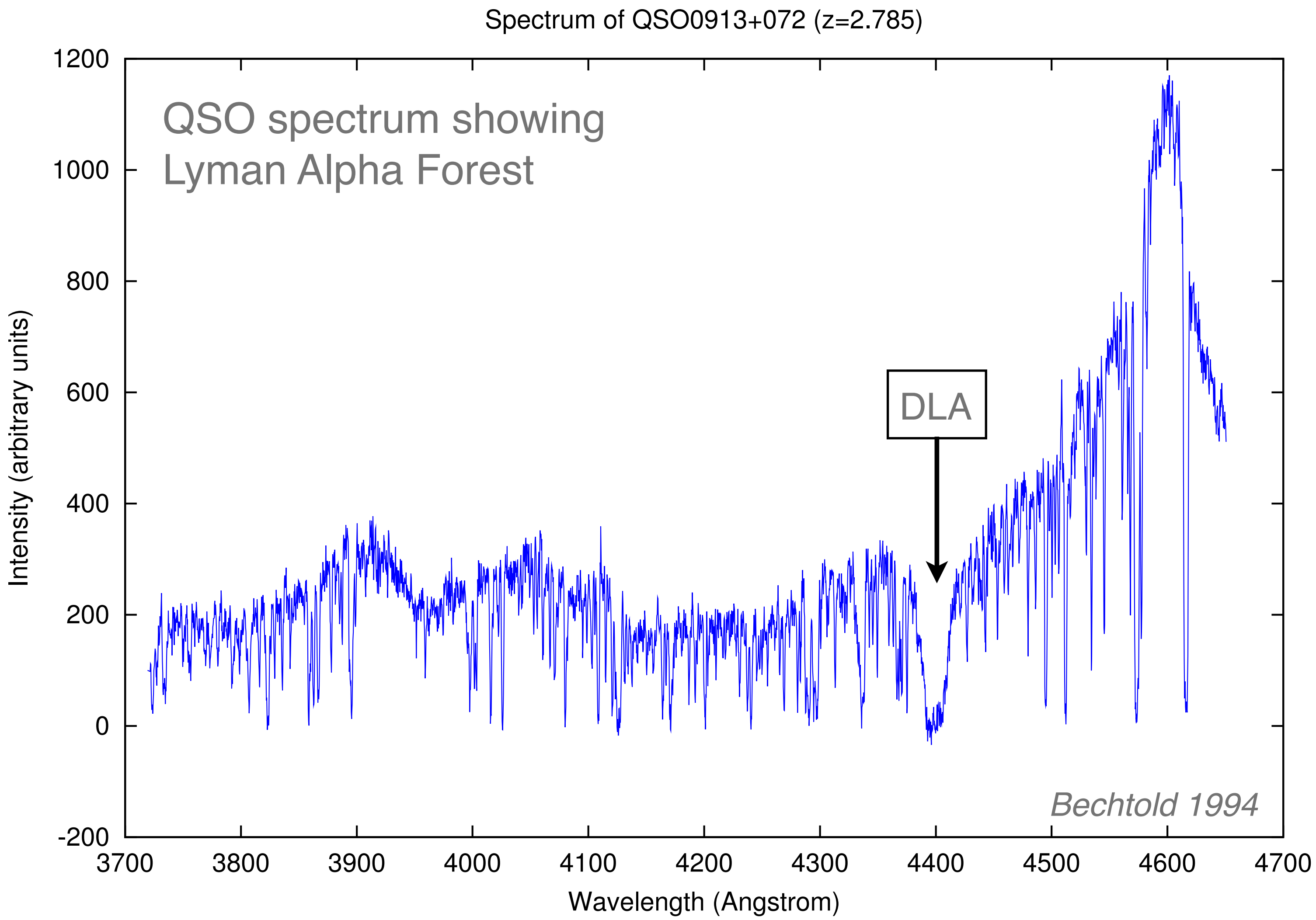
Rotation Measure:

$$RM = \int_0^L n_e B_{\|}\, ds$$

# Intergalactic Medium

ISM of line-of-sight galaxies, gas clouds and the diffuse intergalactic medium can show up in absorption against the radiation of distant galaxies and QSOs

Lyman Alpha provides a strong absorption at 1216 Å in the rest frame of the absorbing gas. Due to cosmological redshift, absorption by different gas clouds in the line of sight occur at different wavelengths ⇒ Lyman Alpha Forest




Clouds with large Hydrogen content (e.g. galaxies) produce deep absorption with damping wings: Damped Lyman Alpha systems (*DLA*s)

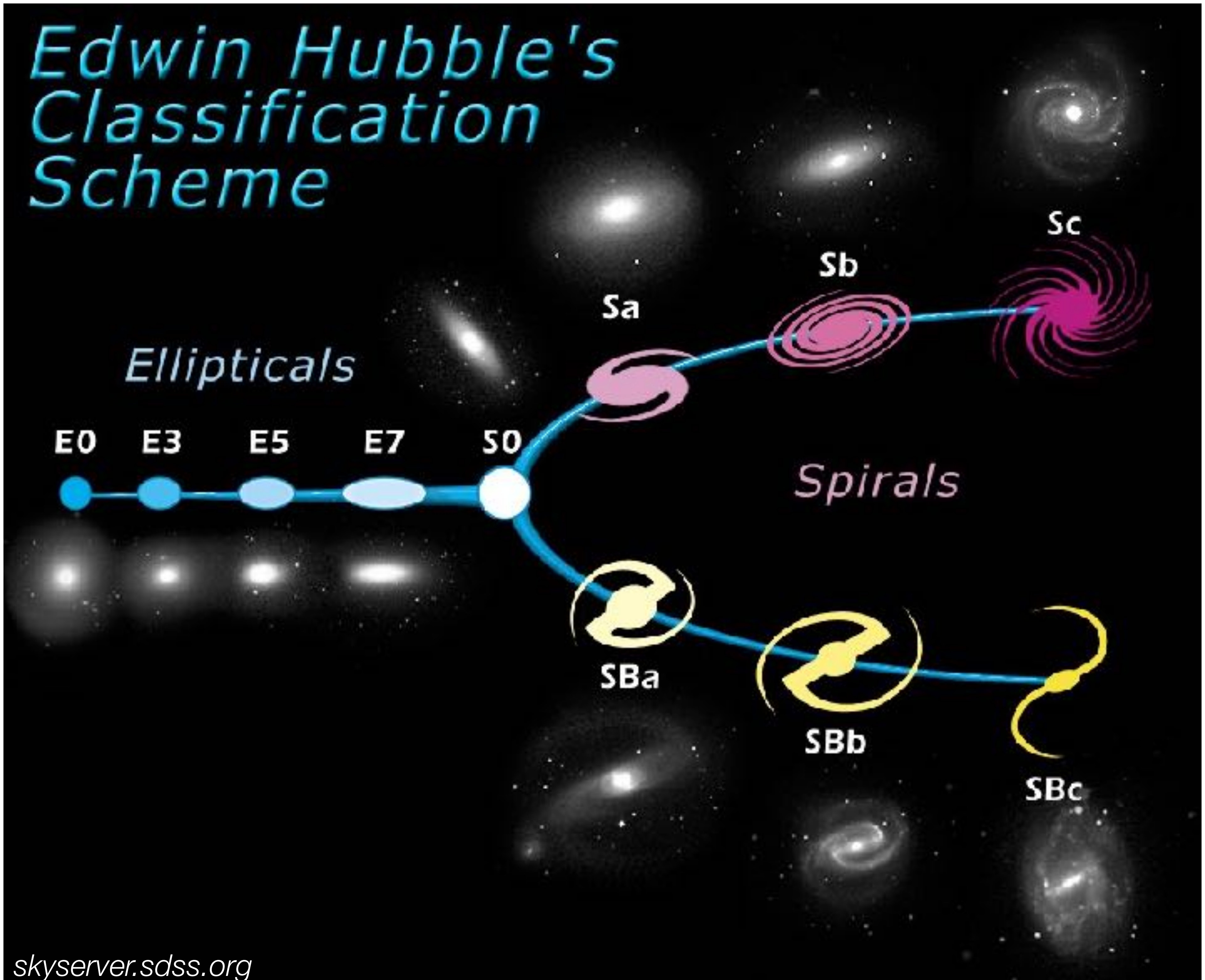
Diffuse Intercloud Medium is almost fully ionized. If not, then radiation shortward of emitted Ly-$\alpha$ would have been completely absorbed: *Gunn-Peterson* effect

# References


- Physical Processes in the Interstellar Medium : *L. Spitzer Jr.*
- The Physical Universe: *F.H. Shu*
- An Invitation to Astrophysics : *T. Padmanabhan*

# Galaxies

# Hubble Classification




Galaxies are the basic building blocks of the universe

Various shapes and sizes:
- flattened **spirals** with high net ang. mom.
- **ellipticals** with lower net ang. mom.
- early galaxies mainly **irregular**

Baryon content $\sim 10^6\ M_\odot$ (dwarfs) to $\sim 10^{12}\ M_\odot$ (giant ellipt.)

Dark Matter
~10-100 x Baryons

# Properties of Galaxies

- Disk galaxies and irregulars are gas-rich, Ellipticals gas poor
- Star formation more prevalent in spirals/irregulars, more old stars in Ellipticals
- More Ellipticals found in galaxy clusters
- Ellipticals grow by merger: giants (cDs) found at centres of rich clusters
- Every galaxy appears to contain a central supermassive black hole
- Correlations:

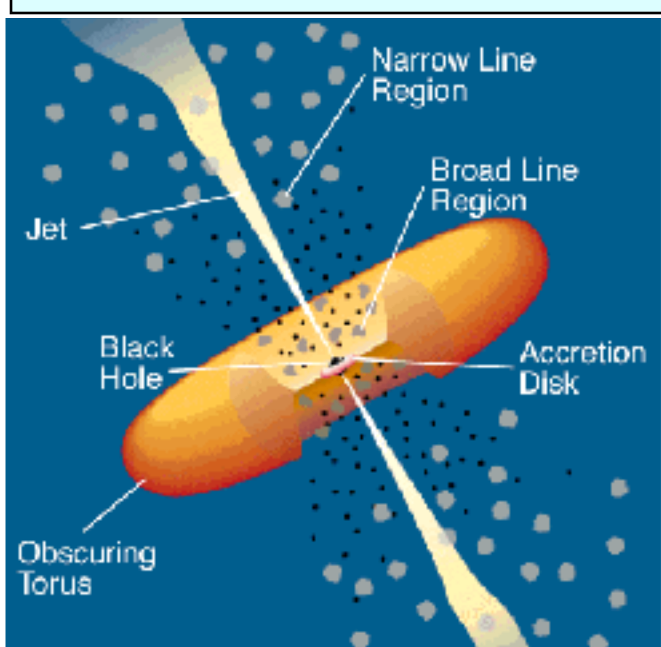
  Ellipticals: *Fundamental Plane*: $R \propto \sigma^{1.4\pm0.15} I^{-0.9\pm0.1}$

  Spirals: *Tully-Fisher relation*: $L \propto W^{\alpha}$

  $\alpha \sim 3-4$ depending on wavelength

  $R$ : size
  $\sigma$ : velocity dispersion
  $I$ : surface brightness
  $L$ : luminosity
  $W$ : rotation velocity

  Central Black Hole Mass: $M_{\rm BH} \propto \sigma^5$, $\sigma =$ velocity dispersion of elliptical galaxy or of central bulge in a spiral galaxy

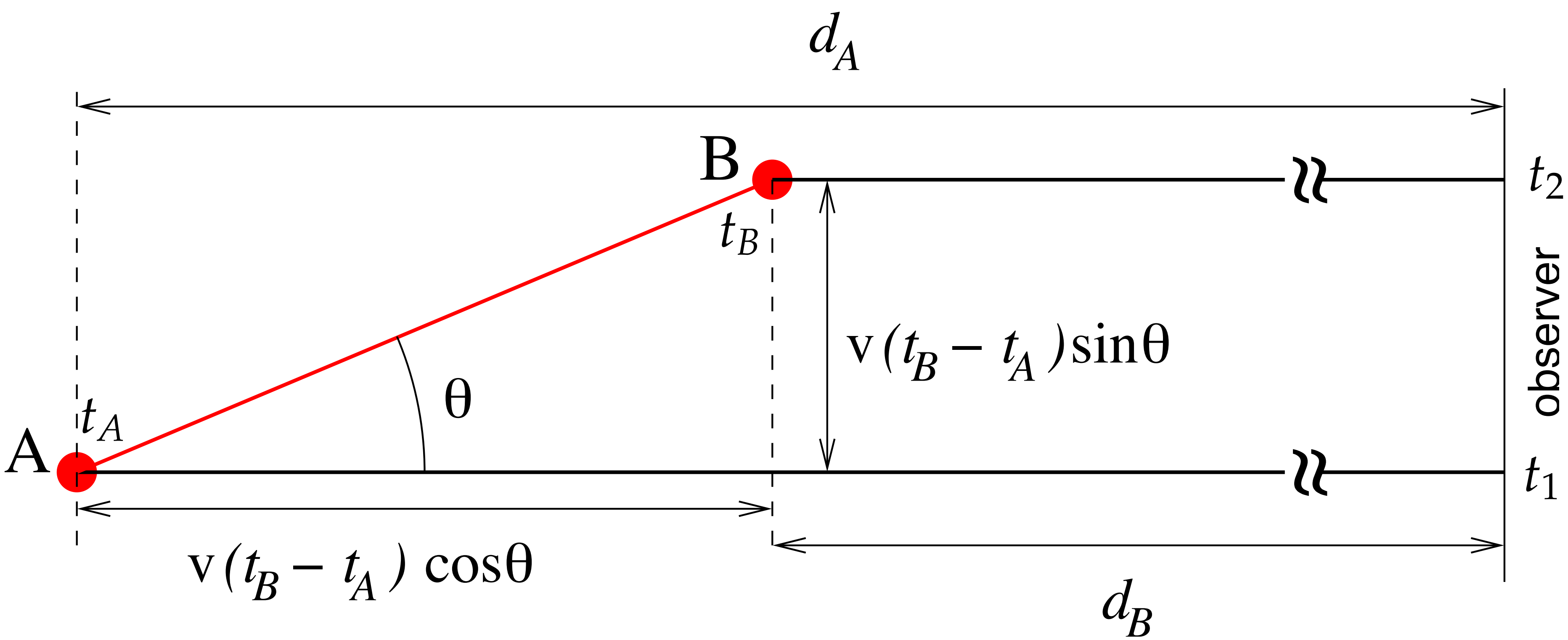



*Urry & Padovani*
heasarc.gsfc.nasa.gov

- If central BH is fed by copious accretion, Active Galactic Nucleus (AGN) results: High nuclear luminosity, relativistic jets, non-thermal emission
  > Broad Line Region, Narrow Line Region, Disk, Torus
  > Diversity of appearance depending on viewing angle & jet strength: Seyferts, Radio Galaxies, QSOs, Blazars, LINERs......
- Emission is variable
- Reverberation mapping of BLR allows measurement of BH mass:
  light echo → $R$ ; spectrum → $v$ ; $M_{\rm BH} = Rv^2/G$

# Superluminal Motion

## Proof of relativistic bulk motion in AGNs



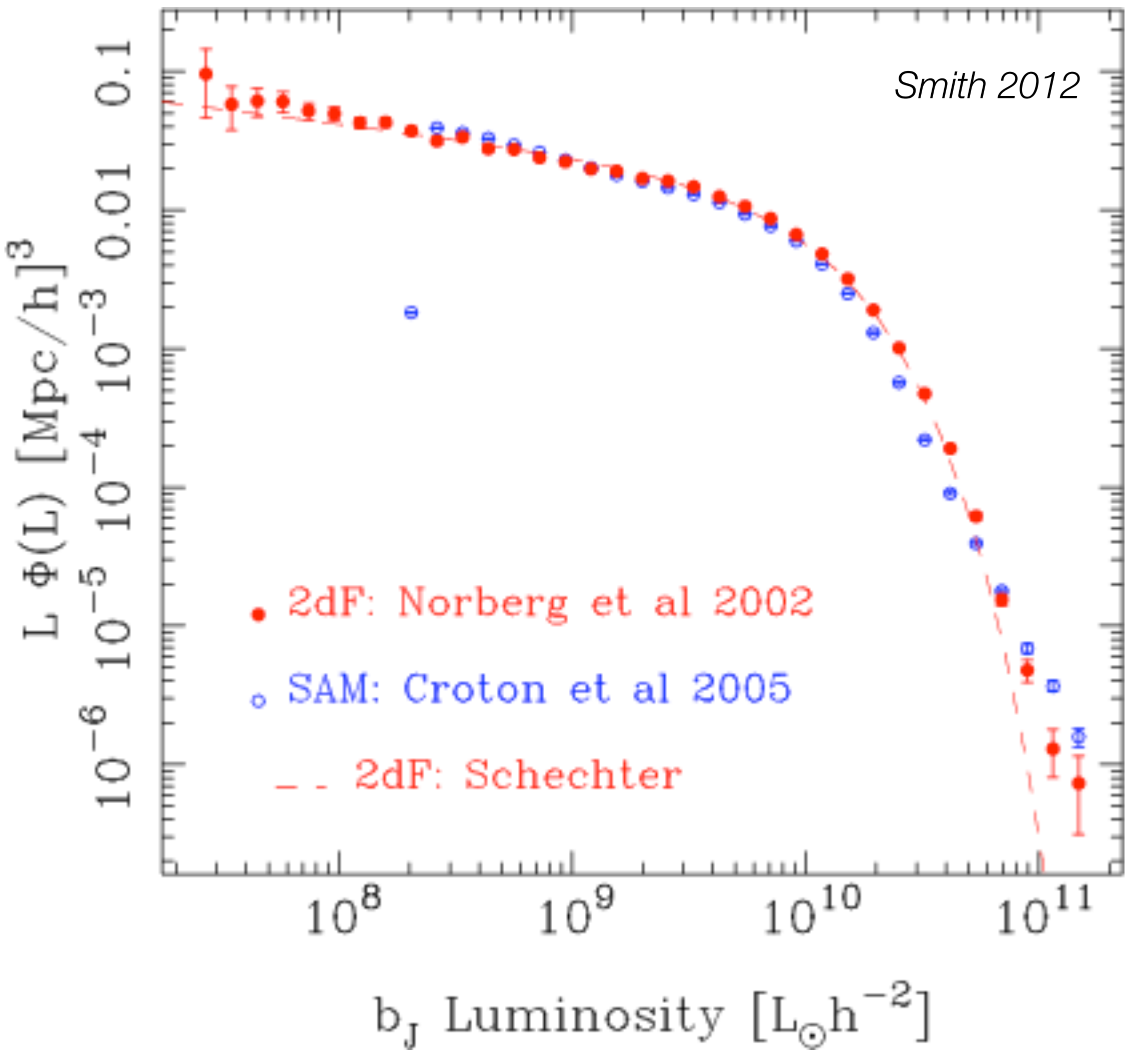

$$(t_2 - t_1) = (t_B - t_A) - \frac{(d_A - d_B)}{c} = (t_B - t_A)\left[1 - \frac{v\cos\theta}{c}\right]$$

$$v_{\rm app} = c\left(\frac{\beta\sin\theta}{1-\beta\sin\theta}\right) \Rightarrow \text{max. } v_{\rm app} = \gamma c\beta \quad \text{at } \cos\theta = \beta$$

faster than light
for $\beta \geq 0.71$

# Galaxy Populations

Luminosity Function



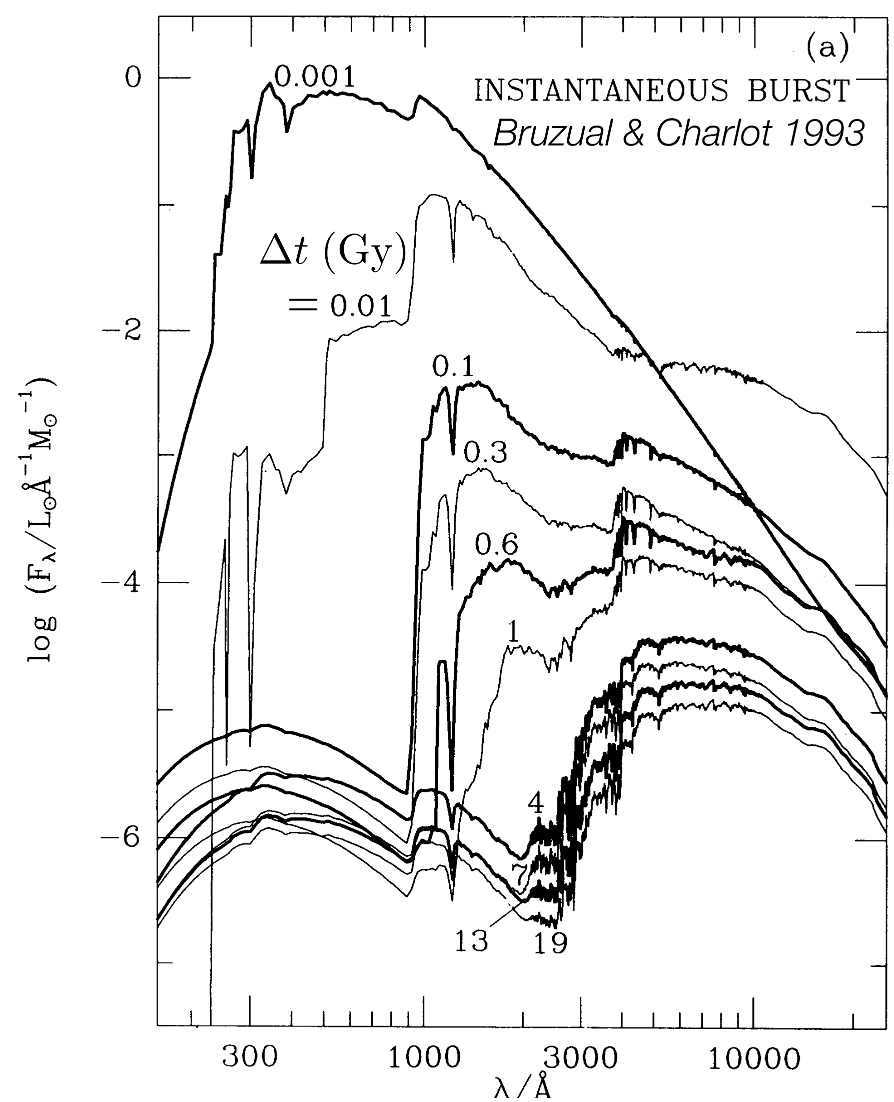

Analytical fit: *Schechter Function*

$$\Phi(L)dL = \Phi_*(L/L_*)^\alpha e^{-(L/L_*)}(dL/L_*)\,, \qquad \alpha \sim -1.25$$

$L_*$ depends on galaxy type and redshift

Luminosity and spectrum of a galaxy would change with time as stellar population evolves

Multiple starburst episodes may be required to characterize a galaxy

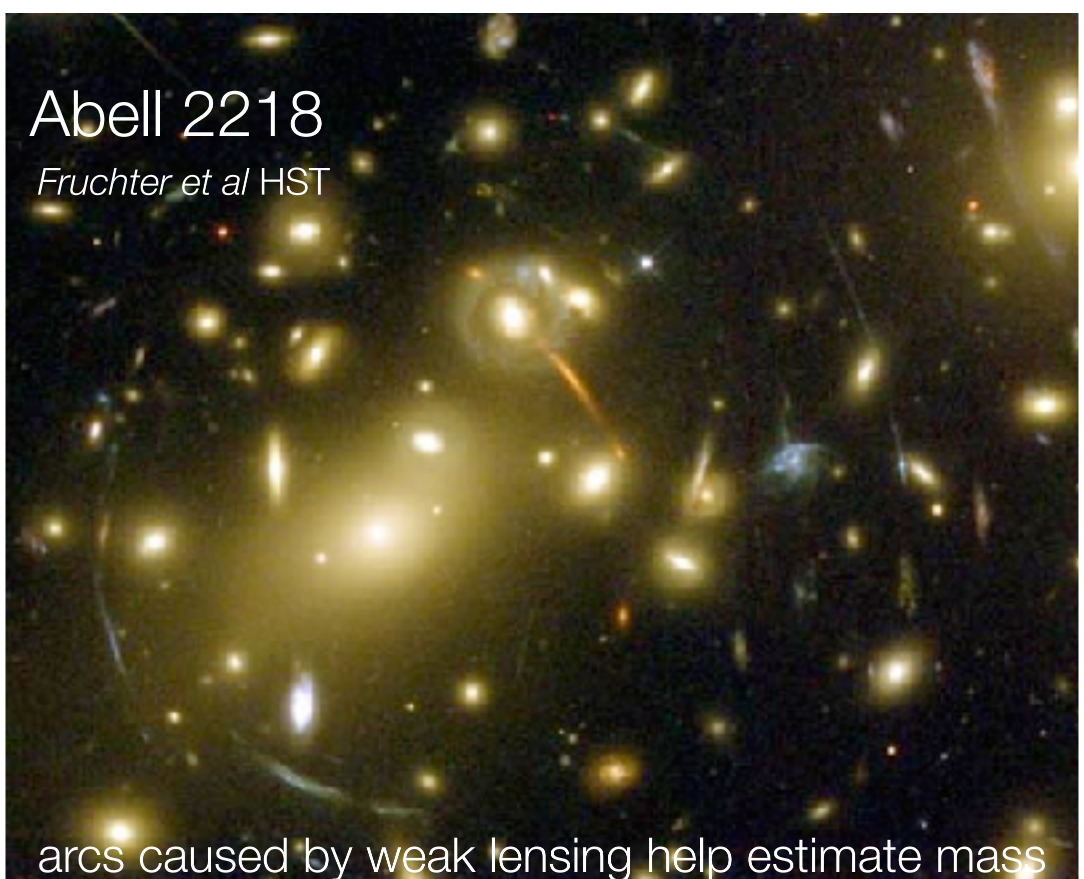



Large fraction of galaxies are found in groups and clusters. Interaction with other galaxies and with the cluster gas can strip a galaxy of gas and terminate star formation

# Galaxy Clusters

- A rich cluster can contain thousands of galaxies bound to a large dark matter halo.
- Diffuse gas in clusters fall into the deep potential well, get heated and emit X-rays.
- Hot gas Compton scatters the cosmic microwave background: Sunyaev-Zeldovich.
- Gravitational lensing can be used to measure cluster mass, revealing dark matter.
- Groups and Clusters grow via collision and mergers.

Density profile of a DM halo: $\rho(r) = \rho_0 / \left[x(1+x)^2\right], \;\; x \equiv r/R_s$ (*NFW*: from simulations)
Virial radius: $\bar{\rho}(r_{\rm vir}) = 200\rho_c(z)$. If hot gas at cluster core has time to cool then it will condense and flow inwards: *Cooling Flow*. Found to be rare: energization by AGN?

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- An Invitation to Astrophysics : *T. Padmanabhan*
- www.astr.ua.edu/keel/galaxies/

# Cosmology

# Hubble Expansion

Over large scales, the Universe is homogeneous and isotropic, and it is expanding. Scale factor $a(t)$ multiplies every coordinate grid.

Let object A receive radiation from object B. The coordinate (comoving) distance between them is $d_c$, which remains constant, and the proper distance is $d = a(t)d_c$. Due to cosmological expansion, the proper distance between these two points increases at a rate $v = \dot{a}(t)d_c$. This is an apparent relative velocity which causes a Doppler shift:

$$\frac{\delta\nu}{\nu} = -\frac{\dot{a}(t)d_c}{c} = -\frac{\dot{a}}{a}\left(\frac{d}{c}\right) = -\frac{\dot{a}}{a}\delta t = -\frac{\delta a}{a}\,,$$ giving $\nu(t)a(t) = \text{constant} \Rightarrow \frac{\lambda_{\rm em}}{\lambda_{\rm obs}} = \frac{a_{\rm em}}{a_{\rm obs}}$

Thus redshift $z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{a_{\rm obs}}{a_{\rm em}} - 1$, or $1 + z = \frac{a_{\rm obs}}{a_{\rm em}} = \frac{a_0}{a(z)}$

subscript 0 denotes a quantity at present epoch

and $v = \left(\frac{\dot{a}}{a}\right)d = Hd$ ; $H \equiv \left(\frac{\dot{a}}{a}\right)$ = Hubble parameter. Present value: $H_0 \simeq 67$ km/s/Mpc

Hubble Time $t_H = \frac{a}{\dot{a}} = \frac{1}{H(t)}$ ~ age of the universe. Present value: $t_{H,0} \approx 14.5$ Gy

# Dynamics of the Universe

Dynamical equations for cosmology follow from a fully relativistic framework. However a Newtonian analogy may be drawn to mimic the basic equations:

per unit mass K.E.+P.E. = const. : $\frac{1}{2}\dot{a}^2 - G\left(\frac{4\pi}{3}\rho a^3\right)\frac{1}{a}$ = const. = $-\frac{1}{2}k$

hence $\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho(t)$ or $H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho(t)$

$\Omega(t) \equiv \rho(t)/\rho_c(t)$

$\Omega(t) = 1 + k/a^2 = 1$ for $k = 0$

In our Universe $k \approx 0$, (flat geometry) *i.e.* $\rho = \rho_c \equiv \frac{3H^2}{8\pi G}$. At present $\rho_{c,0} \approx 0.84 \times 10^{-29}$ g/cm$^3$

Solution of the dynamical equation requires knowledge of $\rho(a)$ : Equation of State

Adiabatic evolution: $P \propto V^{-\gamma} \propto a^{-3\gamma}$ while $P = (\gamma - 1)u = (\gamma - 1)\rho c^2$. Thus $\rho \propto a^{-3\gamma}$

Using this the dynamical equation yields the solution $a \propto t^{\frac{2}{3\gamma}}$ $(\gamma \neq 0)$ flat universe

- For non-relativistic matter $P \propto u_{\rm kin} \ll \rho c^2$, so $\gamma \approx 1$. $\Rightarrow \rho \propto a^{-3} \Rightarrow a \propto t^{\frac{2}{3}}$ matter dominated
- For relativistic matter or radiation $\gamma = 4/3$ $\Rightarrow \rho \propto a^{-4} \Rightarrow a \propto t^{\frac{1}{2}}$ radiation dominated
- For “Dark Energy” $\rho c^2 \approx$ const., *i.e.* $\gamma \approx 0$ $\Rightarrow \rho \propto a^0 \Rightarrow a \propto e^t$ acceleration, inflation

In cosmology the EoS is usually labelled by the parameter $w \equiv (\gamma - 1)$

Present day universe: $\Omega_{\rm tot} \approx 1,\ \Omega_{\rm m} \approx 0.3,\ \Omega_{\rm DE} \approx 0.7,\ \Omega_{\rm b} \approx 0.048,\ \Omega_{\rm rad} \approx 5 \times 10^{-5}$

# Distance measures in Cosmology

Coordinate system: origin at the observer. Comoving (coordinate) distance to a source: $r_c$

Proper distance now: $d = a_0 r_c$, Redshift: $z$ Light leaves source at $t_c$, received at $t_0$

Propagation from $r = r_c$ to $r = 0$ : $-a\,dr = c\,dt$ *i.e.* $-\int_{r_c}^{0} dr = c\int_{t_c}^{t_0} \frac{dt}{a(t)}$

Hence $r_c = c\int_z^0 \frac{1}{a}\frac{dt}{da}\frac{da}{dz}dz = \frac{c}{a_0}\int_0^z \frac{a}{\dot{a}}dz$ and $d = a_0 r_c = c\int_0^z \frac{dz}{H(z)} = c\tau;$ $\tau$ = lookback time $= \int_0^z \frac{dz}{H}$

Luminosity distance $d_L$ :

Received Flux $F \equiv \frac{L}{4\pi d_L^2} = \frac{L}{4\pi d^2}\left(\frac{1}{1+z}\right)\left(\frac{1}{1+z}\right)$ $\therefore\ d_L = d(1+z) = c(1+z)\int_0^z \frac{dz}{H}$

Luminosity

Reduction of photon energy

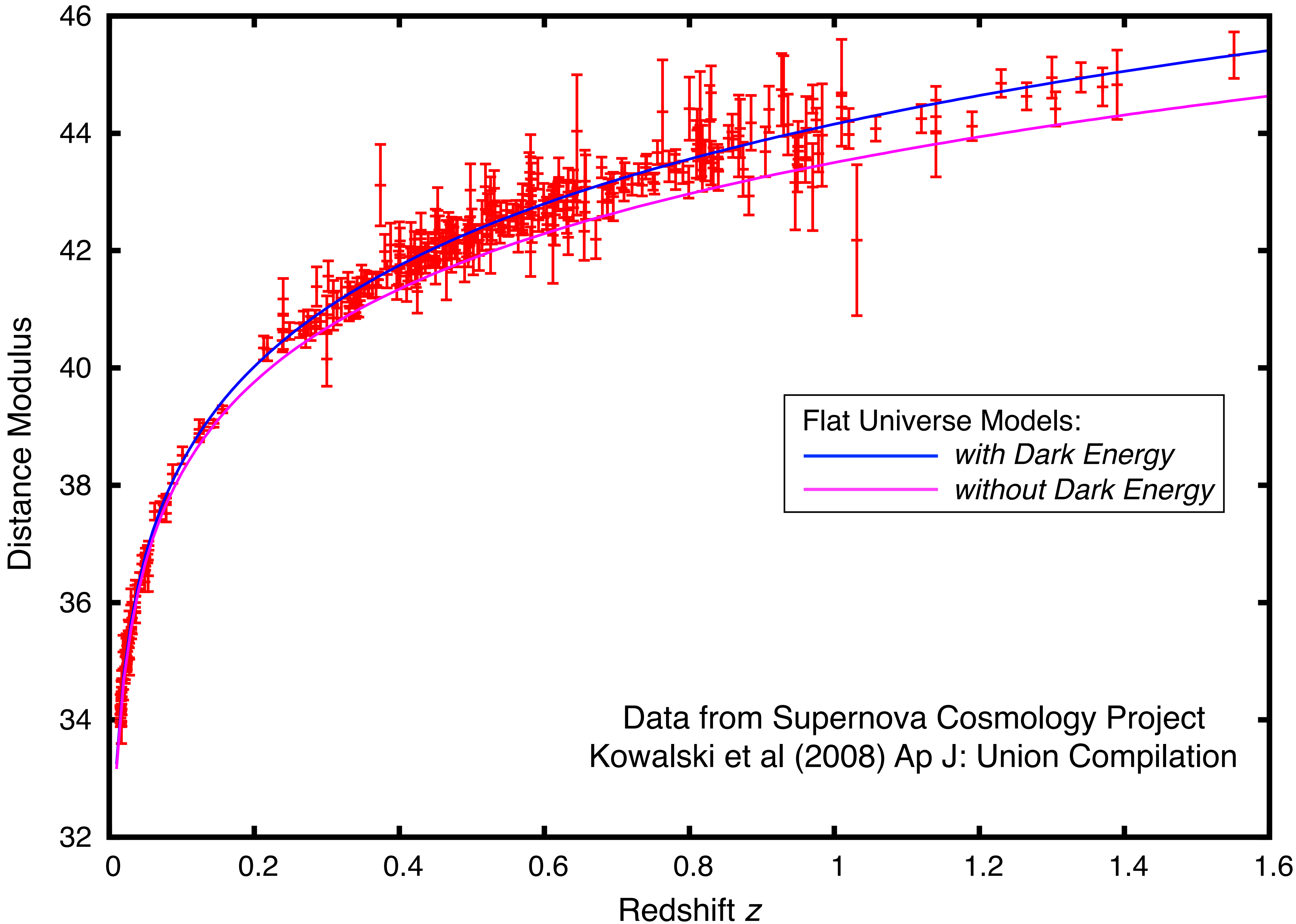
Reduction of photon arrival rate

Angular Diameter distance $d_A$ :

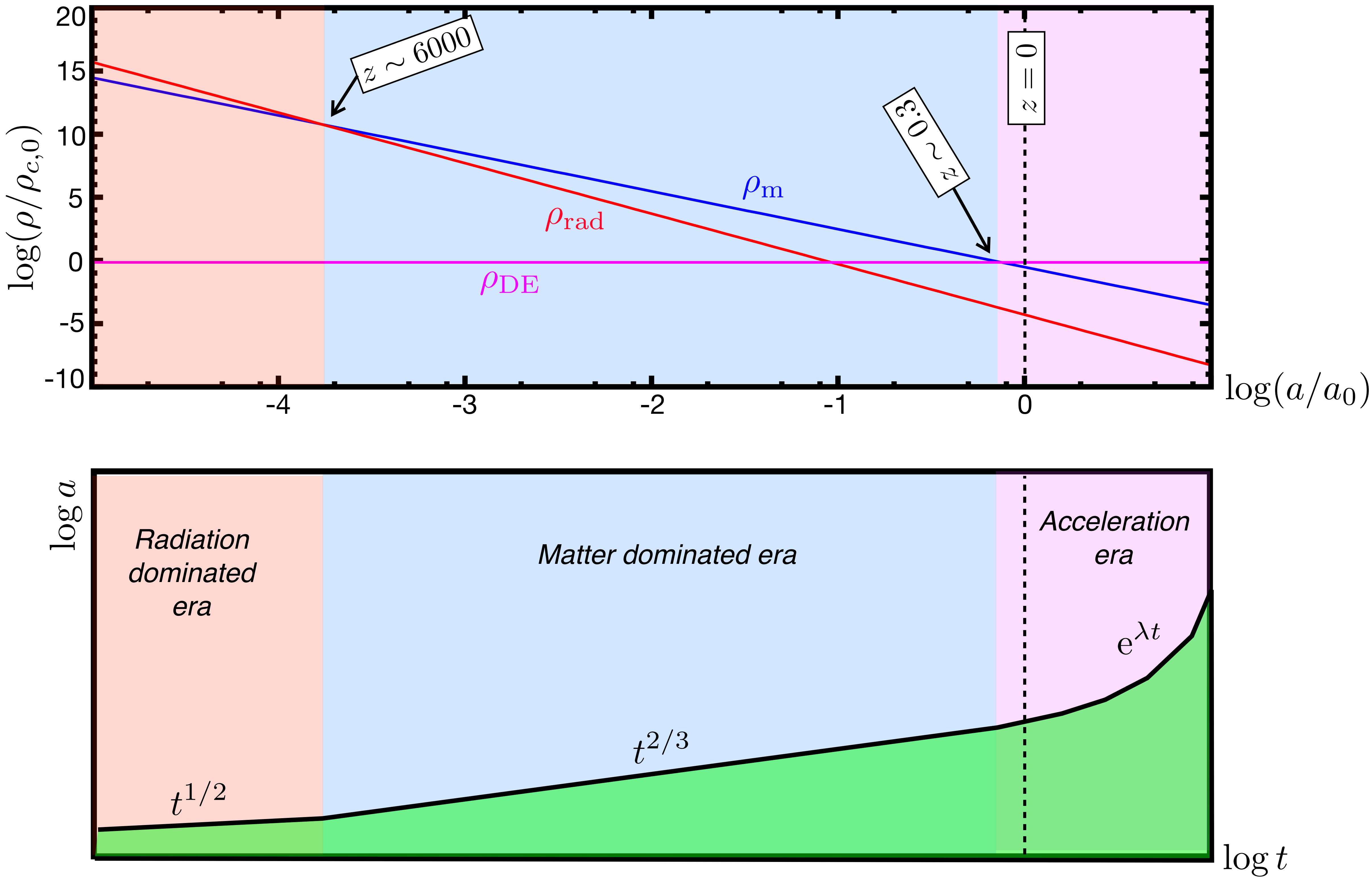
Angular size $\Delta\theta = \frac{\Delta l}{r_c a(t_c)};$ $d_A \equiv \frac{\Delta l}{\Delta\theta} = a(t_c) r_c = \frac{a_0 r_c}{1+z} = \frac{c}{1+z}\int_0^z \frac{dz}{H}$

Transverse linear size

# Supernova Ia Hubble Diagram



# Cosmological Expansion History



# Thermal History of the Universe

Radiation and matter in thermal equilibrium in early universe, at a common temperature
Radiation is Planckian, with blackbody spectrum and energy density $\rho_{\rm rad} \propto T^4$

In cosmological evolution $\rho_{\rm rad} \propto a^{-4}$, thus $T \propto a^{-1}$ ; or $T = 2.73(1+z)$ K
Seen today at microwave bands: *Cosmic Microwave Background Radiation*

If $kT > 2mc^2$ for any particle species, relativistic pairs of the species can be freely created. All relativistic particle species behave similar to radiation in the evolution of their energy density. Total $\rho_{\rm rel}c^2 = \bar{g}a_R T^4$ where $\bar{g}$ =total stat wt of all rel. particle species

In the Early, radiation-dominated universe $a \propto t^{1/2}$. So $t = 1\,{\rm s}\,(kT/1{\rm MeV})^{-2}\,\bar{g}^{-1/2}$
[ $\bar{g} \sim 100$ at $kT > 1$ GeV, ~10 at 1-100 MeV, ~ 3 at < 0.1 MeV ]

Expansion → cooling → pair annihilation of relevant species → energy added to radiation
At $kT \lesssim 1$ GeV, nucleon-antinucleon annihilation → small no. of baryons survived

$\eta = \frac{n_\gamma}{n_{\rm b}} \sim 10^9$ : "entropy per baryon", photon-to-baryon ratio. "Baryon asymmetry" $\sim 10^{-9}$

$kT \lesssim 1$ MeV: $e^{\pm}$ annihilation → $\sim 10^{-9}$ of the pop. left as electrons → charge neutrality
Contents at this stage: neutrons, protons, electrons, neutrinos, dark matter and photons

# Primordial Nucleosynthesis

At $kT > 0.5$ MeV neutrons and protons are in beta equilibrium: $n_\mathrm{n}/n_\mathrm{p} = \exp(-1.3\,\mathrm{MeV}/kT)$

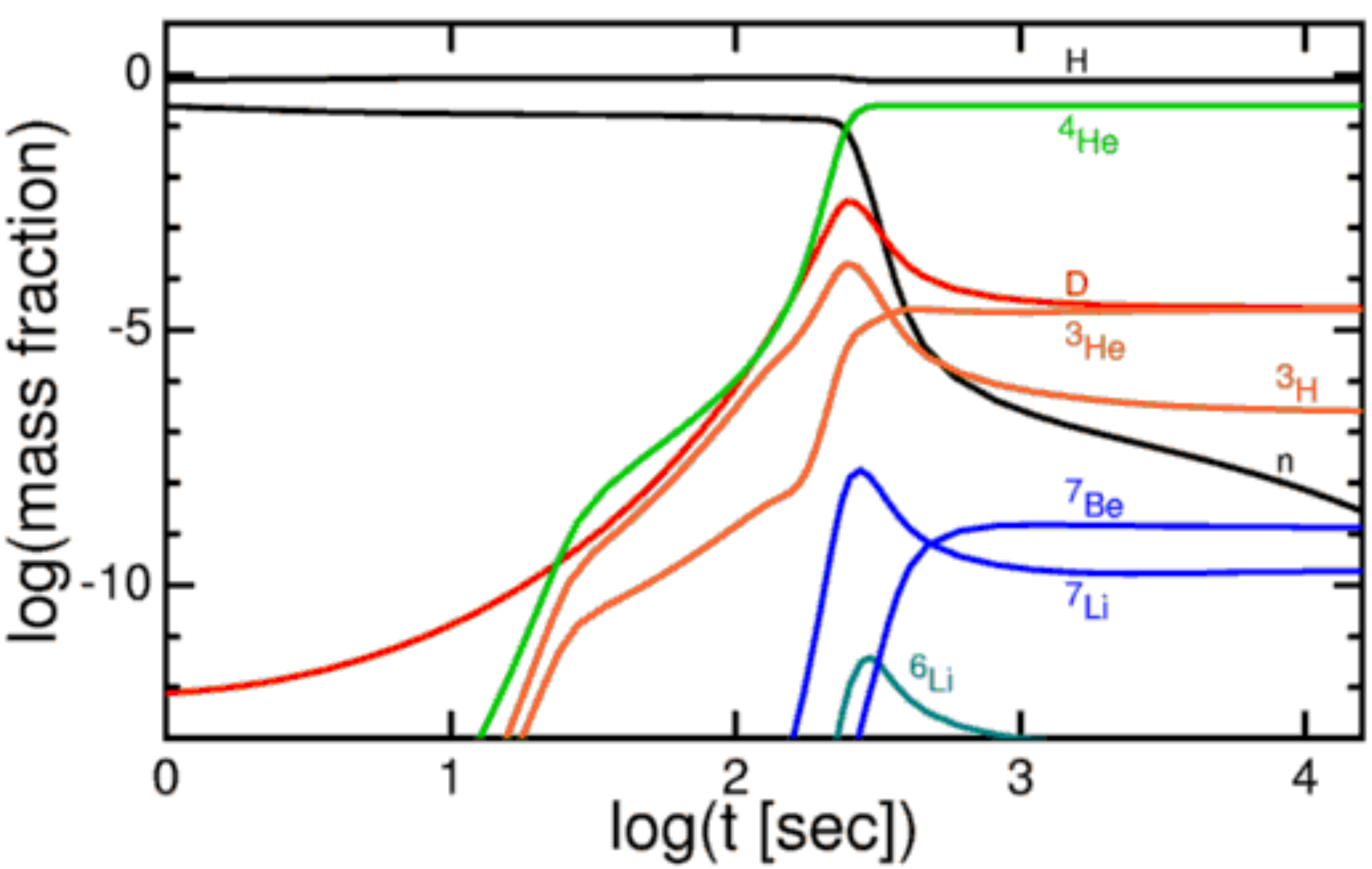
$kT \approx 0.5$ MeV: beta reactions become inefficient, n-p freeze at $n_\mathrm{n}/(n_\mathrm{n}+n_\mathrm{p}) \approx 15\%$

( no n-decay yet as $t_\mathrm{H} << t_\mathrm{decay}$ )

Neutrons will then undergo decay with lifetime of 881 s until locked up in nuclei

Nucleosynthesis begins in earnest only after $kT$ drops to ~0.07 MeV

(decided by the rate of first stage synthesis: that of Deuterium. Most of this $^2D$ then converts to $^4He$)




*http://www.astro.ucla.edu/~wright/BBNS.html*

Neutron fraction drops to ~11% at this point, giving primordial mass fraction:

$^4He$ : ~22%; $^1H$ : ~78%

Synthesis does not progress beyond this stage due to expansion and cooling

The entire nucleosynthesis takes place within approx. the first three minutes after Big Bang: $z \sim 3\times10^8$

# Recombination, CMB, Reionization

After nucleosynthesis the universe has ionized H and He, and electrons. Material is very optically thick due to electron scattering. Cooling continues as $T \propto a^{-1}$. There are also neutrinos, and leptonic Dark matter which do not interact with photons.

Once Dark Matter becomes non-relativistic, gravitational instability develops and self-gravitating collapsed halos start to form. Baryonic matter cannot collapse yet because of strong coupling with radiation.

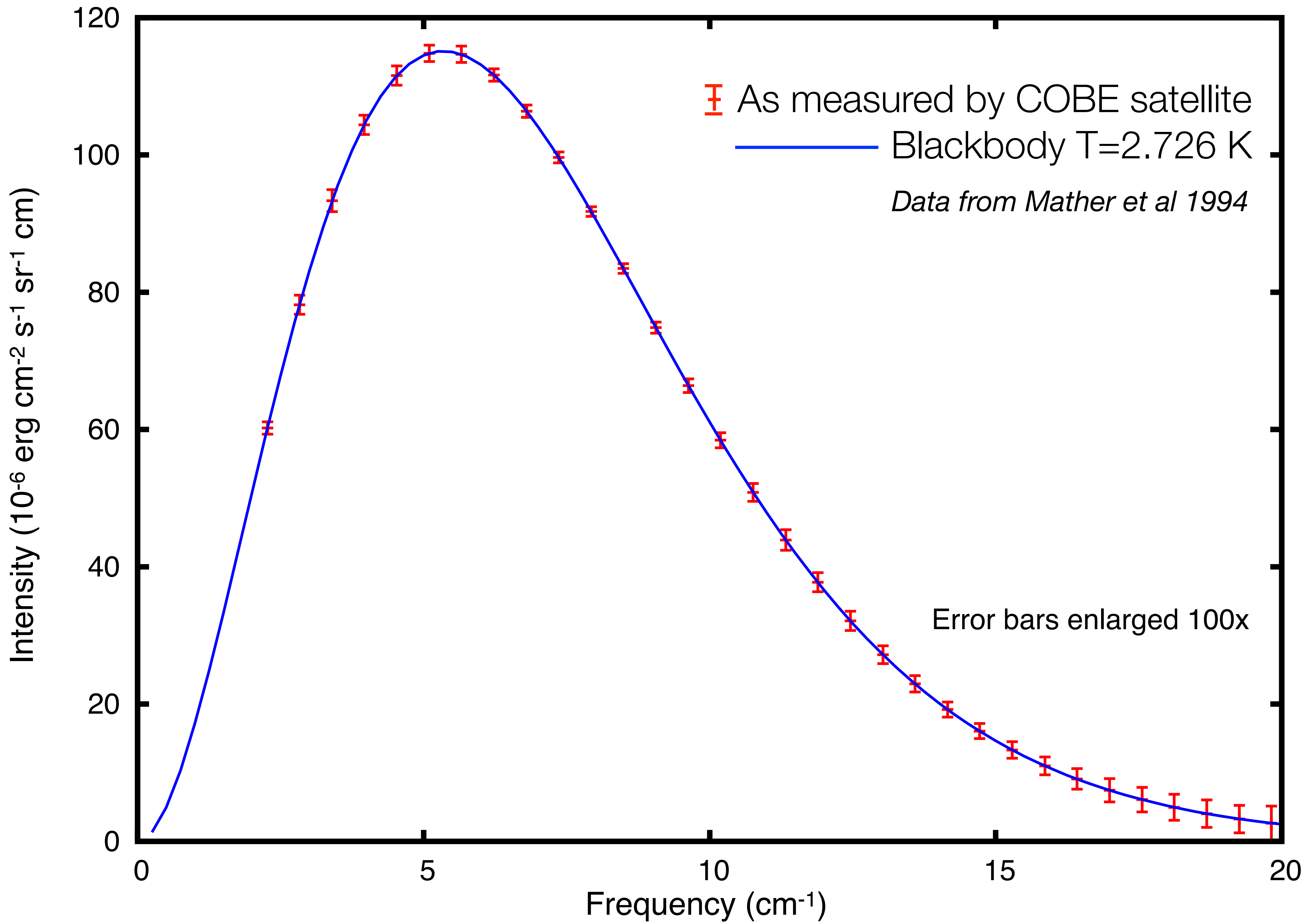
As temperature drops to ~3x$10^3$ K, electrons and ions ***recombine*** to form atoms. Universe becomes transparent to the Cosmic Background radiation. The ***CMB*** we see today comes from this "surface of last scattering" at $z$~1100. Diffuse gas is now neutral.

***Decoupled*** from radiation, baryons now fall into the potential wells already created by Dark Matter, Luminous structures begin to form by $z$~20.

UV radiation from the luminous structures starts ionizing diffuse gas again. These "Stromgren sphere"s grow and overlap, completely ***reionizing*** the diffuse intergalactic medium by $z$~10.

Cosmic Background radiation temperature continues to fall as $1/a$, reaching 2.73 K at present epoch.

# CMB spectrum



# CMB sky distribution

## COBE Satellite Maps

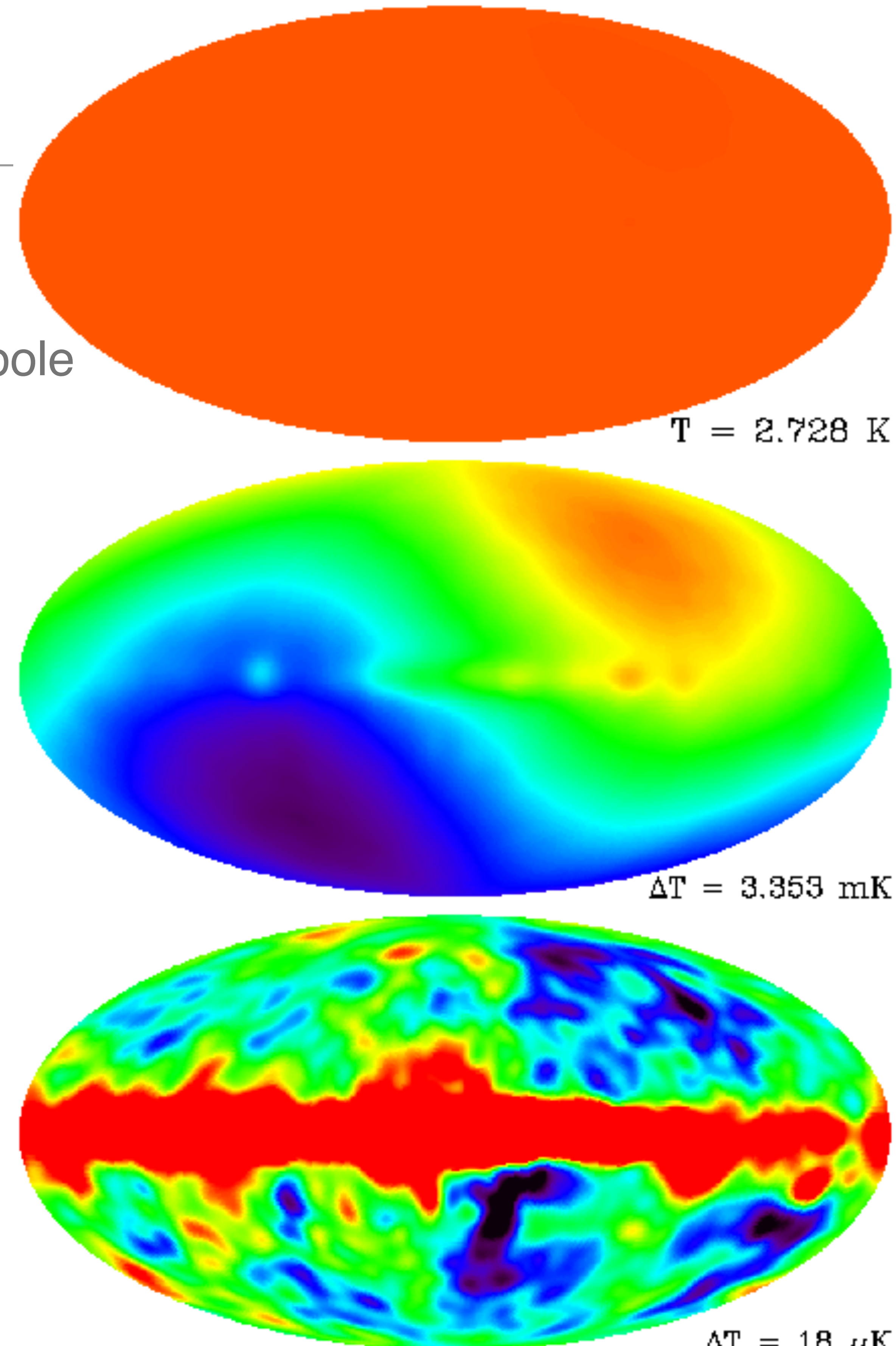
*Bennett et al 1996*

Monopole

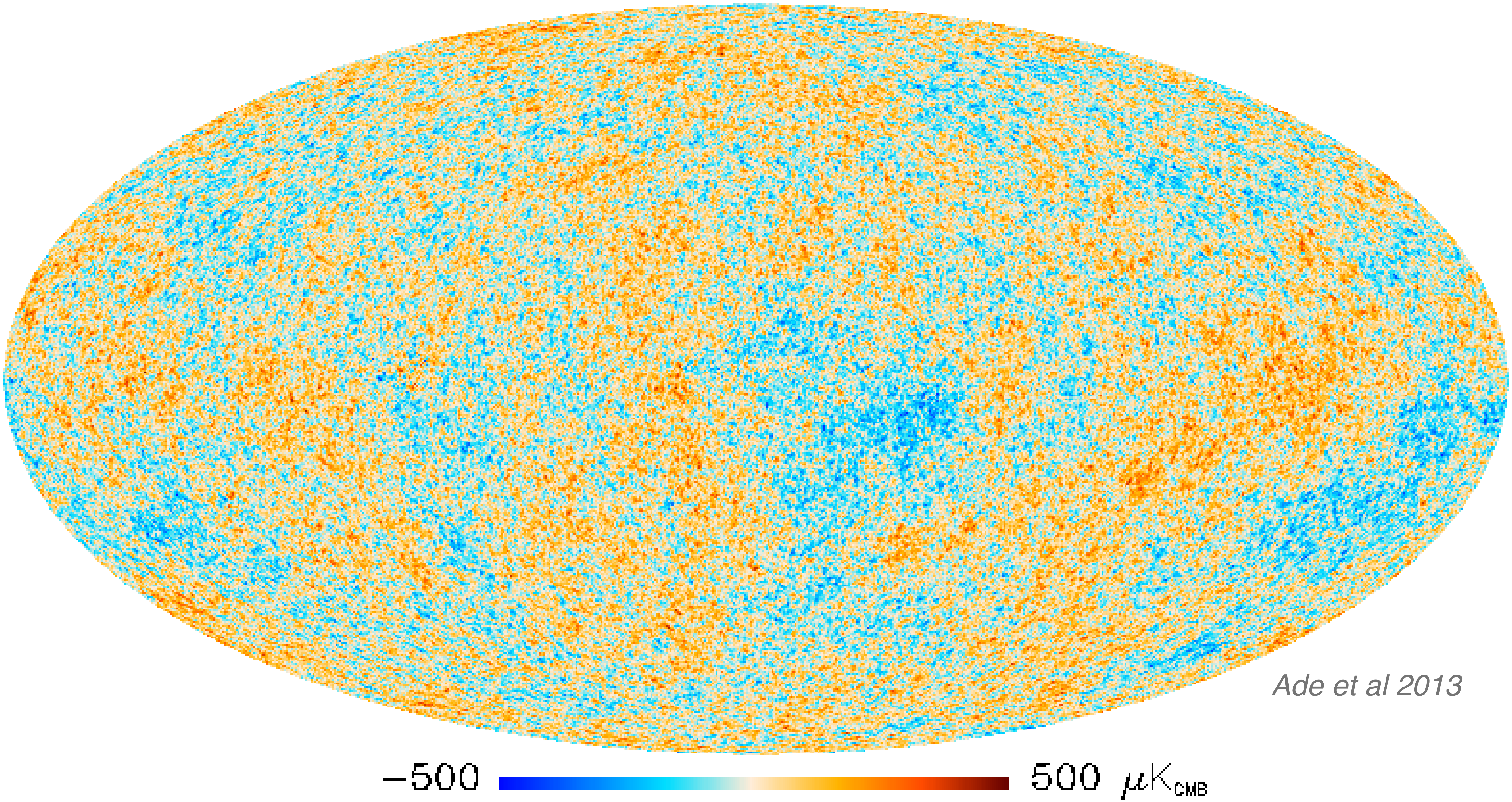
Dipole

*caused by our peculiar velocity*
*~ 370 km/s w.r.t. the Hubble flow*

Higher order anisotropies



# CMB sky distribution




CMB high-order anisotropy map from Planck Satellite

# CMB Anisotropies

**Epoch of Decoupling:**

It follows from Saha equation that the universe recombines when the radiation temperature drops to ~3000 K. This corresponds to a redshift $z_{\rm dec} \simeq 1100$

Defines the last scattering surface (LSS)

In a flat universe $H(z) = H_0 \left[\Omega_{\rm rad,0}(1+z)^4 + \Omega_{\rm m,0}(1+z)^3 + \Omega_{\rm DE,0}\right]^{1/2} \Rightarrow \; H(z_{\rm dec}) \approx 22000\, H_0$

Age of the universe at decoupling $t_{\rm dec} \approx \frac{2}{3}\left(\frac{1}{H(z_{\rm dec})}\right) \approx 4 \times 10^5$ y

CMB anisotropies developed until $t_{\rm dec}$ : *Primary Anisotropy.* Later: *Secondary Anisotropy*

Sources of Primary Anisotropy:

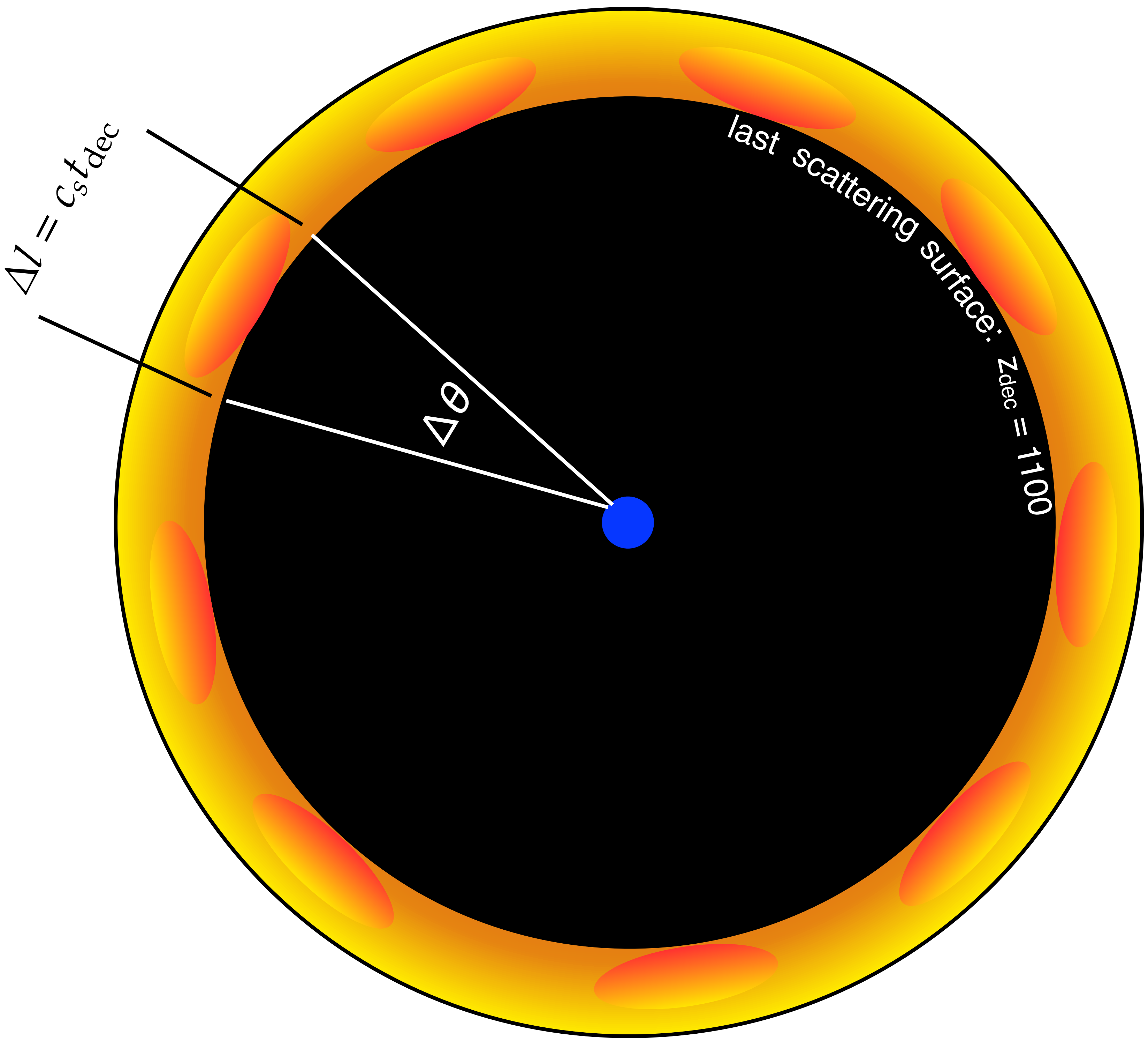
- *Intrinsic, from primordial perturbations*
- *Gravitational Redshift from fluctuating potential at LSS (Sachs-Wolfe effect)*
- *Doppler shifts due to scattering from moving gas*
- *Acoustic oscillations of the photon-baryon fluid*

Modified by:

- *Finite width of LSS*
- *Photon Diffusion (Silk Damping)*

Secondary Anisotropies from: *Integrated Sachs-Wolfe effect, Sunyaev-Zeldovich effect, Gravitational Lensing etc*

# Acoustic Horizon Anisotropy Scale




$$\Delta l = c_s t_{\rm dec} = \frac{c}{\sqrt{3}} t_{\rm dec}$$

$$d_{\rm A} = \frac{c t_{\rm look-back}(z_{\rm dec})}{1 + z_{\rm dec}}$$

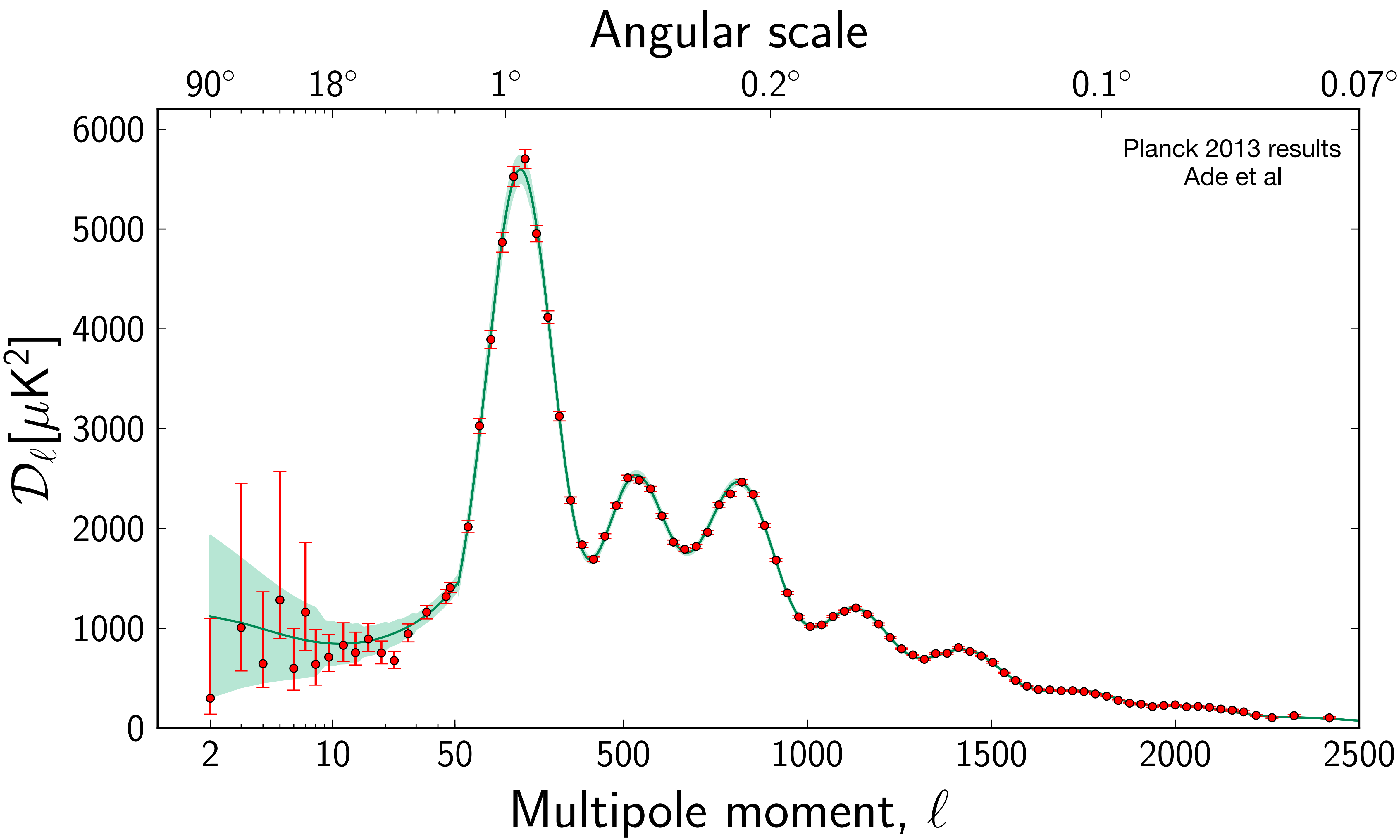
$$\approx \frac{c t_0}{1 + z_{\rm dec}}$$

$$\Delta\theta = \Delta l / d_{\rm A}$$

$$\therefore \Delta\theta = \frac{1 + z_{\rm dec}}{\sqrt{3}} \left( \frac{t_{\rm dec}}{t_0} \right)$$

$$\approx 1^\circ$$

# Anisotropy spectrum of the CMB



# Formation of Structures

Gravitational Instability leads to growth of density perturbations of Dark Matter.

- overdense $[\rho = (1+\delta)\rho_{\rm bg}]$ regions expand less slowly than Hubble flow, eventually stop expanding, turn around and collapse to form halos.
- "critical overdensity" $\delta_{\rm c} \approx 1.68$
- Halo mass distribution at a redshift $z$ : fraction of bound objects with mass $> M$ :

$$f(>M, z) = \mathrm{erfc}\left[\frac{\delta_c(1+z)}{\sqrt{2}\,\sigma_0(M)}\right]$$ (Press-Schechter Formula)

where $\sigma_0(M)$ is the *linearly extrapolated* rms $\delta$ at mass scale $M$ at present epoch

Typical density contrast today at scale $8\,(100/H_0)$ Mpc is $\sigma_8 \approx (0.5 - 0.8)$

Initial density perturbations result from quantum fluctuations.

- However perturbations seen today would require scales much larger than horizon size in the early universe.
- Made possible by accelerated (exponential) growth of $a(t)$ for a brief period: ***inflation***
- Energy provided by a decaying quantum field, which also generates fluctuations

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